DEVELOPING MODELLING BASED MATHEMATICS TEACHING BY MEANS OF THEORIES ON CONCEPTUAL LEARNING

Morten Blomhøj

IMFUFA, NSM, RUC

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1. The issue

Interplay between theory and practice in modelling:
How to integrate theories on modelling and on the learning of mathematics in an in-service course on modelling for teachers at upper secondary level such that the teachers' can use the theories for developing their own practice?

Focus: using theories to support students' learning of mathematics through mathematical modelling.

2. The approach

In the course, the teachers become involved in experimenting with their own teaching practice. They develop modelling projects. In this process we bring theories into play.

Three days seminar: Theory –> develop a modelling project in groups. Go home and try it out (≈10 lessons).

Midterm meeting: Supporting pedagogical observations and the teachers' reporting of their projects.

Two days seminar: Presentation, discussions, reflections.

Publish: dissemination on the internet

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2. The approach

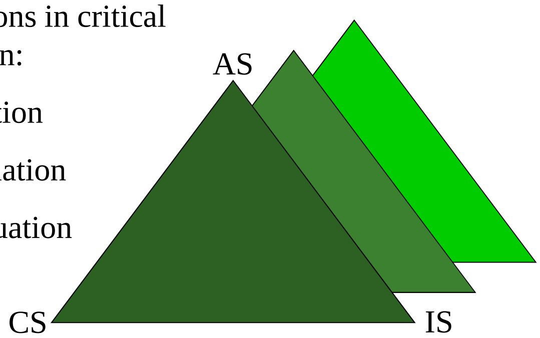
An approach to critical mathematics education with room for collaboration between researchers and teachers:

Three types of situations in critical mathematics education:

CS: The current situation

AS: The arranged situation

IS: The imagined situation



(Skovsmose & Borba, 2004)

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2. The approach

(1) **Experimenting:** Helping the teachers to use theory as a basis for designing and planning their project (AS).

(2) **Analysing** the relation between the actual project (the AS) and our shared ideas about the imagined situation (IS).

(3) **Pedagogical imagining:** Establishing a shared theory based idea about an IS concerning the teachers' modelling project.

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3. An example: A modelling project on the decay of alcohol and THC (cannabis)

Given:

- Data of the decay of alcohol and THC over time
- The amount of alcohol in some popular drinks
- A set of four exercises addressing various aspects of modelling activities – how long time does it take before half of the substance has vanished – is this time depending on the amount of alcohol/THC?
- Modeling your own alcohol consumption at your last party. How long time did it take before the alcohol had vanished according to the model?

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3. An example: A modelling project on the decay of alcohol and THC (cannabis)

Main task:

"Write an article for students of your own age about the decay of alcohol and THC in the human body. In the article you should also explain the mathematics you have used to complete the exercises. Your answers to the exercises and your graphs should be integrated into your article."

The T's learning goals for the students

1. Inquiring relevant questions from their life world
2. Supporting the students' conception of modelling
3. Fostering a critical outlook on mathematical models
4. *Supporting students' learning of linear and exp. functions*
5. *Developing the students' understanding of the parameters in the two models*
6. Train the students to communicate mathematics
7. Supporting the students' IT competences

Modelling

Concept image; linear and exp functions

IT and communication

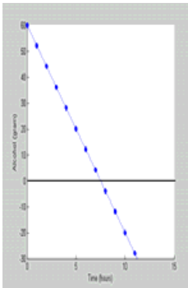
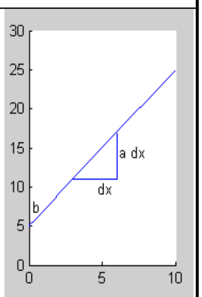
4. Results: The theory – practice relations

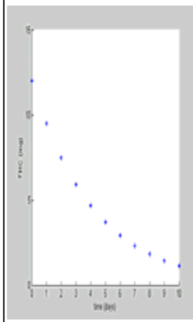
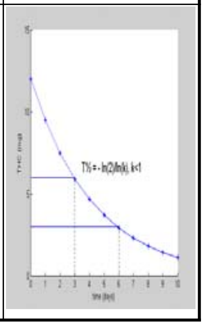
Introduced theory on the learning of mathematical concepts:

- The process-object duality for concept formation, concept image, and the epistemological triangle
- The basic idea that the access to a mathematical concept goes through the meaning of its different representations and their interrelations

For practice – develop a scheme:

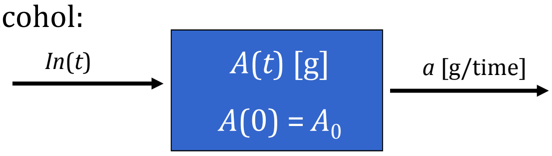
- The theories can be combined to form a representation scheme spanning different representations of the concept of linear and exponential functions. These representations can come into play in the modelling of alcohol and THC.

	Natural language	Numerical	Algebraic	Algorithmic (Excel)	Graphic
Process	8 gram of alcohol is removed by the liver per hour. 12 gram is added per drink (beer) *x* times the slope plus the constant yields *y*. One extra unit of *x* course a change in y of the value of the slope	*x* 0 1 2 *y* 60 52 44 -8 -8 *x* 0 1 2 *y* *b* *a+b* *2a+b* *+a* *+a*	*f(x+1)=f(x)-8* *f(0)=60* $f(x+\Delta x)= f(x) + a\Delta x$; *f(0)= b*	B2=-8 A5=0 A6=A5+1…. B5=60 B6=B5+$B$2 …. Can be generalised by change of parameter and initial value	
Object	After five drinks and *x* hours: *y* =60 – 8*x* gram of alcohol is left in the body A linear combination with constant sum.	*x* 0 1 2 3 *y* 60 52 44 36 A tabel of (*x*,*y*) with *y = ax+b*	*y = -8x +60* *f*(*x*) = *ax* + *b*	B2=-8; B3=60; A5=0 A6=A5+1…. B5=$B$2*A5+$B$3 B6=$B$2*A6+$B$3 ……. The algorithm is general due to parameters.	

	Natural language	Numerical	Algebraic	Algorithmic (Excel)	Graphic
Process	For THC the con-start half life is 3 days in the body. 0.79 of the THC is present after one day. In a exponential decay the rate of change is a factor of the amount. If x increases 1unit, y is multiplied with k (k<1).	x 0 1 2 y 12 9.5 7.5 ·0.79 ·0.79 x 0 1 2 y b kb k^2b $\cdot k$ $\cdot k$	$f(x+1)= 0.79 \cdot f(x)$; $f(0)=12$ mg $f(x+\Delta x)= k^{\Delta x} \cdot f(x)$ $f(0)=b$	B2=0,79 A5=0 A6=A5+1…. B5=12 B6=B5 · \$B\$2 …. Can be generalised by change of parameter and initial value	
Object	12 mg to start with, after x days $12 \cdot 0.79^x$ mg is left in the body. A decreasing exponential function.	x 0 1 2 3 y 12 9.5 7.5 6 A tabel of (x,y) with $y = b \cdot k^x$	$y = 12 \cdot 0.79^x$ $f(x) = b \cdot k^x = b \cdot e^{\ln(k)\cdot x}$; $T_{½} = -\ln(2)/\ln(k)$, k<1	B2=0,79; B3=12; A5=0 A6=A5+1…. B5=\$B\$3 · \$B\$2^A5 B6=\$B\$3 · \$B\$2^A6……. The algorithm is general due to parameters.	

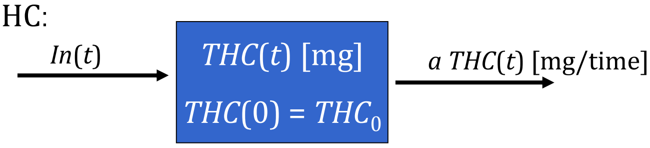
4. Results: Compartment diagrams helps students to model dynamical systems and to learn mathematics

For alcohol:



If $In(t)$=0, $A'(t) = - a$ => $A(t) = A_0 - a\,t$

For THC:



If $In(t)$=0, $THC'(t) = - a\ THC(t)$ => $THC(t) = THC_0\, e^{(-at)}$

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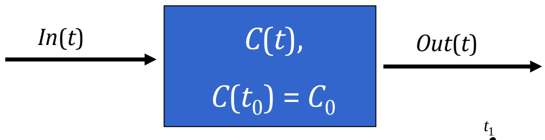
4. Results: Compartment diagrams helps students to model dynamical systems and to learn mathematics

In general:



$$C'(t) = In(t) - Out(t); C_0 = C(t_0) \qquad C(t_1) = C_0 + \int_{t_0}^{t_1} In(t) - Out(t)\, dt$$

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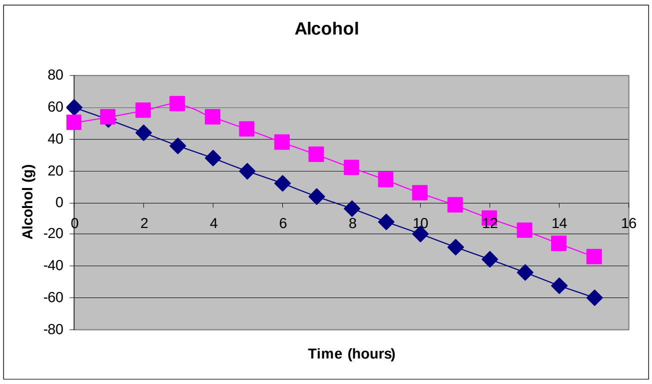
4. Results: Modelling the decay of alcohol



Some groups extended the model: The figure shows the decay model for alcohol with a initial value of 60 g (equal to 5 beers) and a model with 48 g as the initial value and one additional drink each of the following three hours.

Excel model

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4. Results: The theory-practice relation

- The process and object aspects of the representations can be interpreted and be given meaning in the modelling context
- The scheme can function as a tool for facilitating the teachers' use of theory in their practice
- The teachers were introduced to the learning potentials spanned in the scheme at the first seminar
- However, the way the scene was set for the students' modelling work in the project did not systematically challenge the students to work with all the different representations.
- In the final seminar the teachers reflected about the unfulfilled learning potentials in the project. Suggestions for how to guide the students to work with the different representations in the scheme the next time were discussed.

4. Results: The students' learning and reflections

- The students gained some experience with modelling.
- The students interpreted their results and investigated questions based on their own experiences.
- The students used different representations for setting up and analysing their models.
- The students were able to reflect about and criticize the model assumptions.
- The comparison of the two models (for alcohol and THC) made it possible for the students to develop their understanding of the different representations.
- The spreadsheet representation of their models made it possible for the students to develop their models and to investigate their behavior.

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