Ownership and alienation in mathematical group work activity

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The wind chill factor
If you ever have waited in a bus stop or taken a walk during a cold winter day then you know that it feels colder when the wind blows. This cooling sensation is caused by the combined effect of temperature and wind, and is known as wind chill. Living in a country where winters are cold can be hazardous to the public’s health. People die from over-exposure to the cold and many more suffer injuries resulting from hypothermia and frostbite. Therefore, it is important to be able to calculate the wind chill with certain degree of accuracy. Your group’s task is to come up with a mathematical model to calculate the wind chill. Once you have your model, you have to argue how you would test it and how you would refine it.
How did you feel when reading the wind chill task?
How do you think your students would feel if they were given this task?
I am interested...

(1) In understanding the role of emotions in mathematical activity, particularly during group work.

(2) If and how practitioners can use emotions to foster effective collaborative work.
Usually emotions are seen as antagonist of human rationality and clear thinking.

Traditionally, there is a dualist view about cognition and emotions as opposed and contradictory (body vs mind).

Vygotsky (1896 – 1934) and his students and colleagues (Leont’ev, Luria and others) were devoted to overcoming this dualistic perspective. They initiated what is now known as Cultural-historical Activity Theory.
Emotions from a cultural-historical activity theory perspective

Emotions are those psychological functions that signalise the personal sense of one’s own and others’ actions thereby making emotions constitutive for activity regulation as a whole (Leont’ev, 1978).

Emotions are defined through their function in the activity:

**Appraisal function**
Appraising a situation according to its relevance for the satisfaction of motives.

**Activity regulating function**
An emotion manifests itself in an urge to act or an urge to inhibit an action.
For example, anger mobilises a subject in order to threaten or even attack his or her opponent who is perceived as intentionally blocking the goal striving of the subject.
Emotions are linked inseparably to cognition because they contain a schema-based appraisal of the situations, determining the quality of the various emotions (Ratner, 2000). Emotions involve concepts of the self and entail moral and ethical dimensions (Radford 2015).

“Emotions are feelings that accompany thinking. They are the feeling side of thoughts; thought-filled feelings; thoughtful feelings” (Ratner, 2000, p.6).

Emotions can be, but not have to be, linked to expressive reactions and bodily reactions.
Emotions from a cultural-historical activity theory perspective

An analysis of the cultural character of emotions require elucidating how social relations of practical cultural activity and their associated concepts imbue emotions with specific characteristics, form the experiences which socialise emotions in people’s minds and bodies, are motivated by emotions and dictate a unique relationship between emotions and psychological processes. Hence, the members of a culture also acquire the dominant concepts about emotions during the process of internalisation. This also enables them to display and feel the appropriate emotions in their social activities.
Emotions in group work activity

- Second year materials engineering module. Students are accepted with only a GCSE mathematics, and those with A-level mathematics usually come with a B or even a C.

- Wind chill modelling coursework worth 5% of the module’s assessment.

- Four weeks to produce a presentation, given in front of their peers.
Emotions in group work activity

- We follow three groups of four students each. Report here on two of them.

- Video recording and observational notes of fourteen project meetings during three weeks, outside of lectures (once in a pub, but generally in the library).

- We were interested in tracking their “emotional trajectories”, and how these helped or hindered progress.
Emotions in group work activity

GROUP 1.

- They had 6 meetings in total. The first three meetings were spent trying to work out the problem and deciding the variables and the constants of the possible model.

- This led to feelings of frustration when the objectives set by the group were clearly not met.

- This frustration also emanated from their inability to use tools to make sense of the problem.

Student1: Oh God, MATLAB! Do we dare dable?
Student2: I got 95% in the coursework...
Student3: Took me ages...
Student2: We did this all in the first year, and then I’ve no idea how to do it. I think we need to think about kind of what everything... what all these things mean. We’re still not really anywhere close to figuring how.
Emotions in group work activity

GROUP 1.

- Frustration turned sometimes into anger at comments that were made by group members that were interpreted as not helping to advance a solution to the problem.

Student1: We spent four weeks but have a model that doesn’t work. How is the presentation going (to Student3 and Student4)?
Well, here is something that doesn’t work.
Student2: It’s difficult so no shame in failing.
Student1: *(In anger)* Yeah, but he (the lecturer) gave us the skills.
Emotions in group work activity

GROUP 1.

- This frustration had important consequences for the group’s dynamics, for example:
  - When one student’s ideas were constantly dismissed, and he opted not to participate in the discussion any more.
  - But other times, it led to humour and progress.

Student2: Let’s be magicians!
Student1: Magicians, mathematicians and look at the greek alphabet.
Student2: That’s why I said kappa.
Student1: What’s kappa?
Student3: I don’t know what’s kappa.
Student4: Kappa it’s a sports brand, isn’t it?
Student2: No! It’s a greek letter.
Student1: Can’t we go with miu or something like that?
Student2: Are we going to find the least hipster greek letter possible? You see, it’s a big frog as well.
Emotions in group work activity

GROUP 1.

- Unable to achieve sufficient understanding, the feeling is one of desperation, and eventually resignation with a solution which they are not entirely satisfied with.

Student2: We found it somewhere, I guess is in the word document.
Student1: Let’s do some actual maths.
Student2: It’s taken us nearly three weeks to do some maths!
Student3: I wish I understood maths.

Student1: I don't know how this will work. I don't know what I'm doing.
Student2: I don't like it, it's a shot in the dark.
Student4: That's why it's only 5%.
Student1: Frustrating. My brains will be fried.
GROUP 1.

“Everything should be as simple as it can be, but not simpler” – Albert Einstein.
Emotions in group work activity

GROUP 1.
Emotions in group work activity

GROUP 1.

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A graph illustrating the relationship between wind speed and the perceived difference in temperature:

\[ y = 0.2x + 0.04 \]
Emotions in group work activity

GROUP 2.

- They had 5 meetings in total, but the last two were dedicated at developing the presentation and rehearsing it.

- There is a feeling of confidence about the task, and their conversations were a mix of “job-talk” and satisfaction at being “efficient” in the task.

(Talking about doctoral students in engineering)
Student1: We did 5 years worth in two hours.

Student2: It’s only 5% of the module. Think carefully how much effort you put.
Emotions in group work activity

GROUP 2.

➢ The discourse is conciliatory, and the feeling is one of comradery and easiness.

Student1: Which one is $x$? It just seems linear in a different way.
Student3: Yeah. It does. Is a linear I’m happier with.
Student2: Well sorry for not producing the proper graph for you (laughs).
Student4: Both have their merits for what they are looking at.
Student1: Good say!

➢ Even mistakes are dealt in a “mature” way, which creates a feeling of being relaxed.

Student2: Forget what I just said, it sounds stupid!
Emotions in group work activity

GROUP 2.
- They are more willing to (at least) discuss doing things that Group 1 did not.

Student2: Let’s do an experiment.
Student1: On campus.
Student2: Does anyone have a heat camera?

Student1: We can make it 3-D. *(But the program returned an application error!)*
Student3: Just do it in Excel.
Emotions in group work activity

**GROUP 2.**

- This practical approach meant they easily took for granted things from “reliable” sources (e.g. Environment Canada)

<table>
<thead>
<tr>
<th>Student1:</th>
<th>They were their approximate, and that was a basic estimate but I’m not sure what that estimate is based on.</th>
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<tbody>
<tr>
<td>Student4:</td>
<td>Doesn’t really matter, just plot them.</td>
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<tr>
<td>Student2:</td>
<td>Have a look, just to have an idea of what kind of graph, it it’s linear or not, which probably isn’t.</td>
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<tr>
<td>Student1:</td>
<td>Of course it’s never linear; nothing’s linear.</td>
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Emotions in group work activity

GROUP 2.

The ‘Feels Like’ Temperature with Regards to Different Wind Speeds at Various Actual Temperatures

- 10km/h $y = 1.2x - 57.6$
- 20km/h $y = 1.2606x - 76.382$
- 30km/h $y = 1.3042x - 89.513$
- 40km/h $y = 1.3491x - 102.45$
- 50km/h $y = 1.36x - 106.48$
- 60km/h $y = 1.3709x - 110.31$
Emotions in group work activity

GROUP 2.

\[ y = mx + c \]

\[ T_f = \frac{\delta T_f}{\delta T} T + c \]

\[ T_f = (-0.0000661W^2 + 0.00805W + 1.125)T + (0.0201241W^2 - 2.45082W - 35.1462) \]
Emotions in group work activity

GROUP 2.

![Wind Speed Gradient with Regards to Wind Speed](image1)

![Y-Intercept Values with Regards to Wind Speed](image2)
The emotional journeys of both groups were very different, and that led to different results.

Both solutions were unsatisfactory in mathematical terms.

Group 1 was filled with frustration and anger, but it seems that at the end those feelings kept pushing them to try to make sense of the problem.

Group 2 was filled with comradery and easiness but they couldn’t come up with a substantially better solution than Group 1.

However, Group 2 was much more efficient: less meetings, and a more collaborative effort. Arguably, a better disposition towards maths.
**Alienation and ownership in mathematical modelling**

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<tr>
<th>Alienation</th>
<th>Ownership</th>
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<tbody>
<tr>
<td>There is alienation when there is no understanding of the mathematics. Students are not responsible beyond assessment.</td>
<td>But there is ownership when there is “re-invention”; students own the mathematics they “create”</td>
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<td>There is alienation when students adjust their efforts according to marks given for coursework.</td>
<td>But here is ownership when the mathematics has a visible, tangible use.</td>
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Mathematical modelling can promote ownership by making mathematics useful for the development of “intellectual labour power”.
Why then is mathematical modelling (the type that produces ownership) the exception rather than the norm at school and (allegedly?) in undergraduate education?