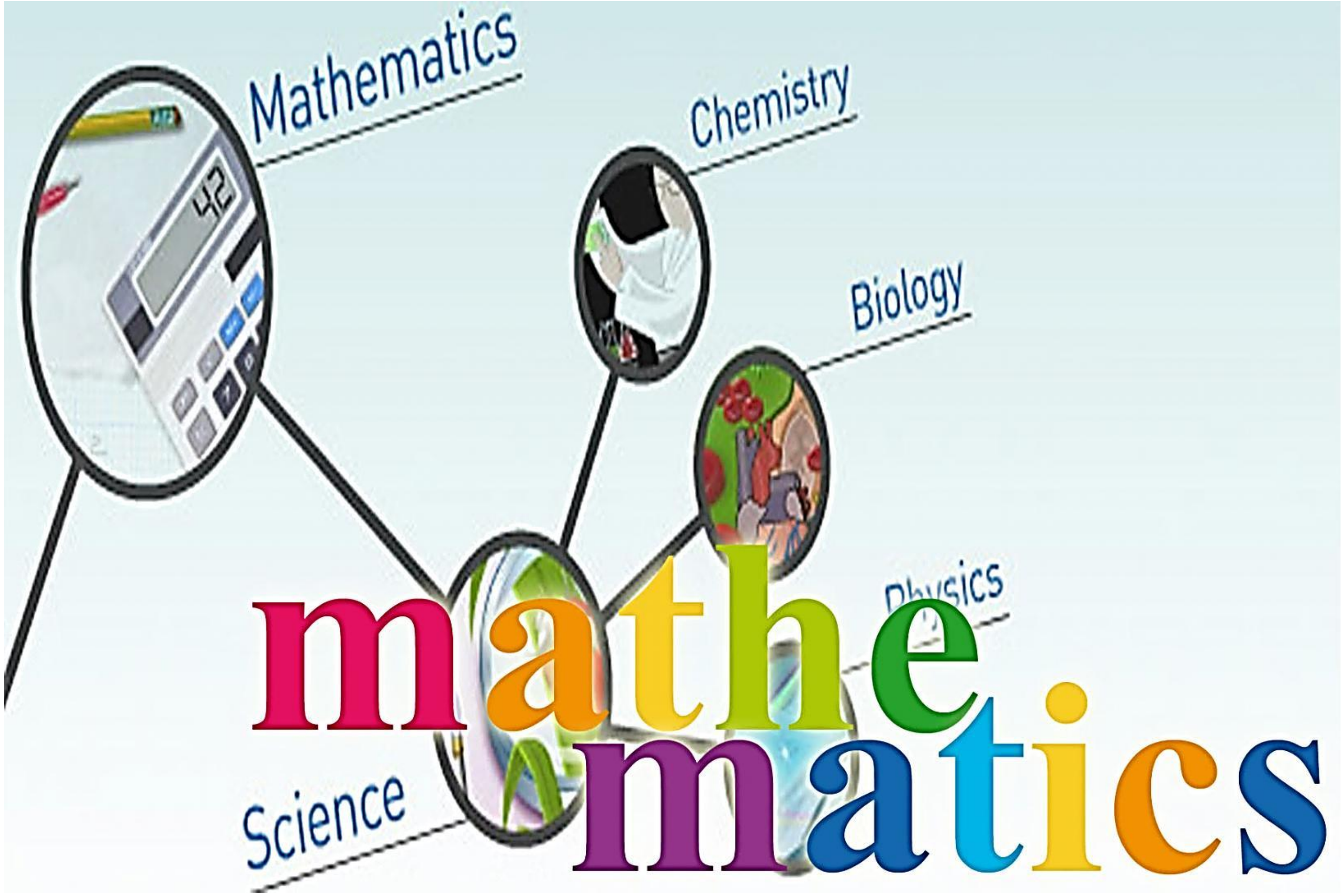
Modeling in an Integrated Physics and Mathematics Course

Ángeles Domínguez

Department of Mathematics

Tecnologico de Monterrey

Tecnológico de Monterrey



Tecnológico de Monterrey

Knowledge as "plaza mall"

Structure

- Mexico – Tecnologico de Monterrey
- The case of physics and mathematics
- Our approach
- Academic achievement
- Modes and representations
- Model Application Activity
- Model Construction – Final project
- Curricular sequence

MEXICO
Tecnologico de Monterrey

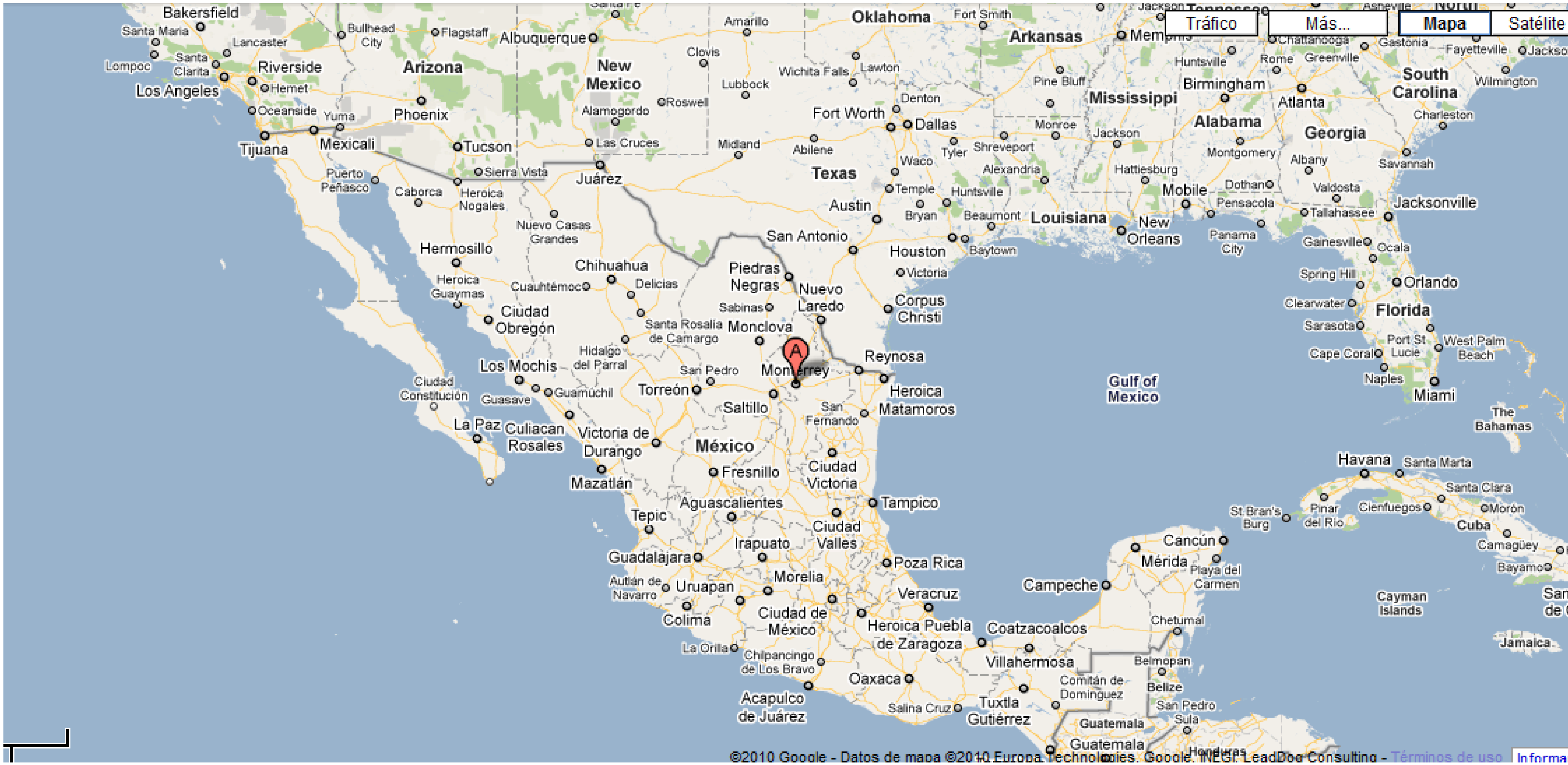
Mexico

- **Population:** 122 millions (approximately)

- **Area:** 2,000,000 km^2 (approximately)

- **Most important cities:**
  - Mexico City (21 millions)
  - Guadalajara (5 millions)
  - Monterrey (4.5 millions)

Monterrey



Tecnológico de Monterrey

ITESM SYSTEM www.itesm.mx/

- Multi-campii private university
  - 30 campii in Mexico
  - 42 high schools

Educational system:
1 year kindergarten
6 years elementary school
3 years middle school
3 years high school

- **Number of students:** 50,000
- **Programs:** 57 undergraduate programs, 53 master programs, and 10 doctoral programs

Campus Monterrey

- Number of students: 12,000 approximately
- 5 high schools (10º, 11º y 12º)
- On-line courses
  - Graduate programs
  - Professional development programs

Integrated Physics and Mathematics Course

The case of physics and math

"The rules that describe nature seem to be mathematical"

Richard Feynman

Feynman, R. (1998). The Meaning Of It All: Thoughts Of A Citizen-scientist.

Motivation

- **Bridge a gap** between Physics and Mathematics through the curricular design of the integrated courses.
- **Physical concepts** are used to study the physical phenomena represented by models.
- **Models** that go through iterative cycles of conjecturing, testing and revising, until they satisfy constrains and provide a feasible explanation of the phenomenon under certain assumptions.
- **Models and modeling perspective** reflects the **physicist's way of understanding the world**,
  so we should teach physics that way.

Redesign of Calculus Courses

- Our first attempt to close that gap was a redesign of the mathematical curriculum.
  - Use of physical phenomena to explore mathematical functions
- However, the math and physics courses were still taught separately, and often occurred that when the physics course needed some math concepts, in the mathematics course that material had not been covered yet.

Background - Integrated courses

- SUCCEED-Coalition → Freshman Year Engineering Curriculum Improvements at North Carolina State → Engineering and computing
- Integrating Calculus and Introductory Science at Louisiana Tech University → An improvement in student connections between science and mathematics course content
- Integrated Quantitative Science → University of Richmond Integrated Science Course for STEM Majors (chemistry, biology, computer science, physics, and math)
- IMPaCT → **I**ntegrated **M**athematics, **P**hysics and **C**ommunication **T**rack in the Engineering Curriculum at University of Miami
- Integrating Science and Math into the Freshman → University of Hartford → Principles of Engineering, Physics I and Calculus II

Background

- Design and implementation of an integrated Physics and Mathematics Course for first semester engineering students.
- Promote connections between physics and mathematics.
- Develop in students a more positive attitude towards science and engineering.
- Develop skills such as: critical thinking, creativity, collaborative work and autonomy.

Frameworks

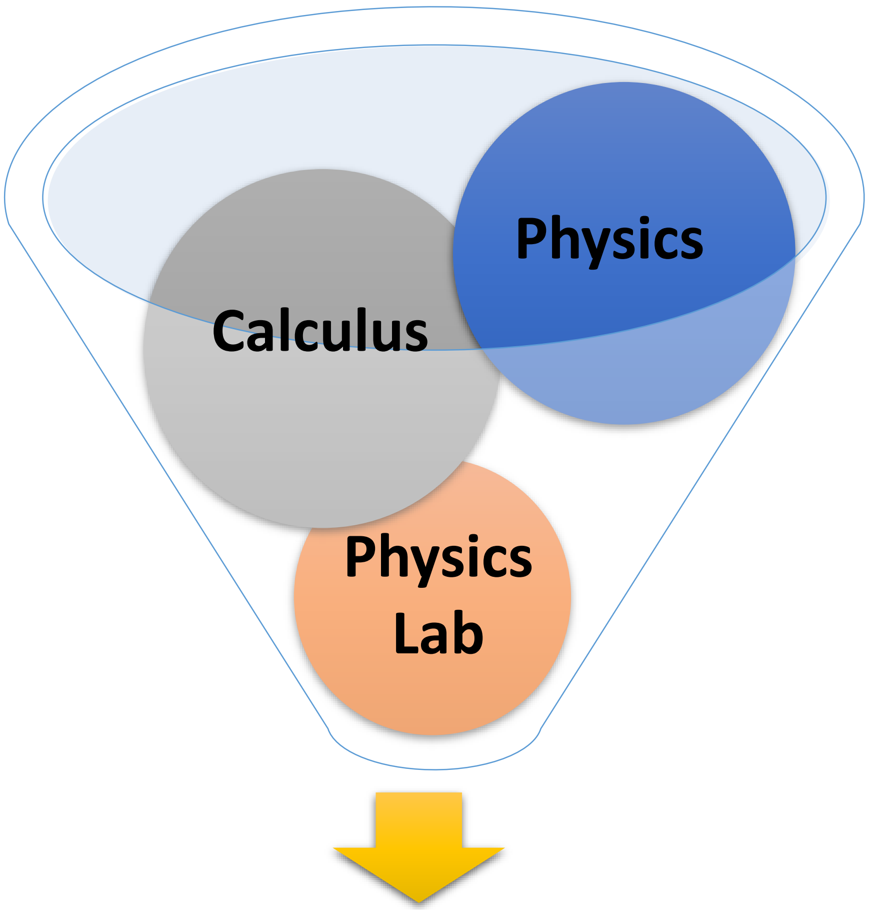
- **Hestenes** - Modeling Instruction - students' building the concepts with activities and real life investigations.
- **Lesh and Doerr** propose a foundation on models and modeling perspective on Mathematics teaching, learning, and problem solving.

Our approach: Objectives

FIS – MAT 1 Physics 1 + Mathematics 1
for Engineering students

a) Improve students' connections between Physics and Math,
b) Increase students' motivation towards engineering, and
c) Develop diverse competences such as critical and creative thinking, and collaborative work

Integrated course: Fis-Mat 1



Integrated Physics and Mathematics Course (Fis-Mat 1)

Physics 1
+ Calculus 1
Physics Lab

7.5 hr/week

Courses for Engineering students

Tecnológico de Monterrey

Integrated course: Fis-Mat 1

Physics content

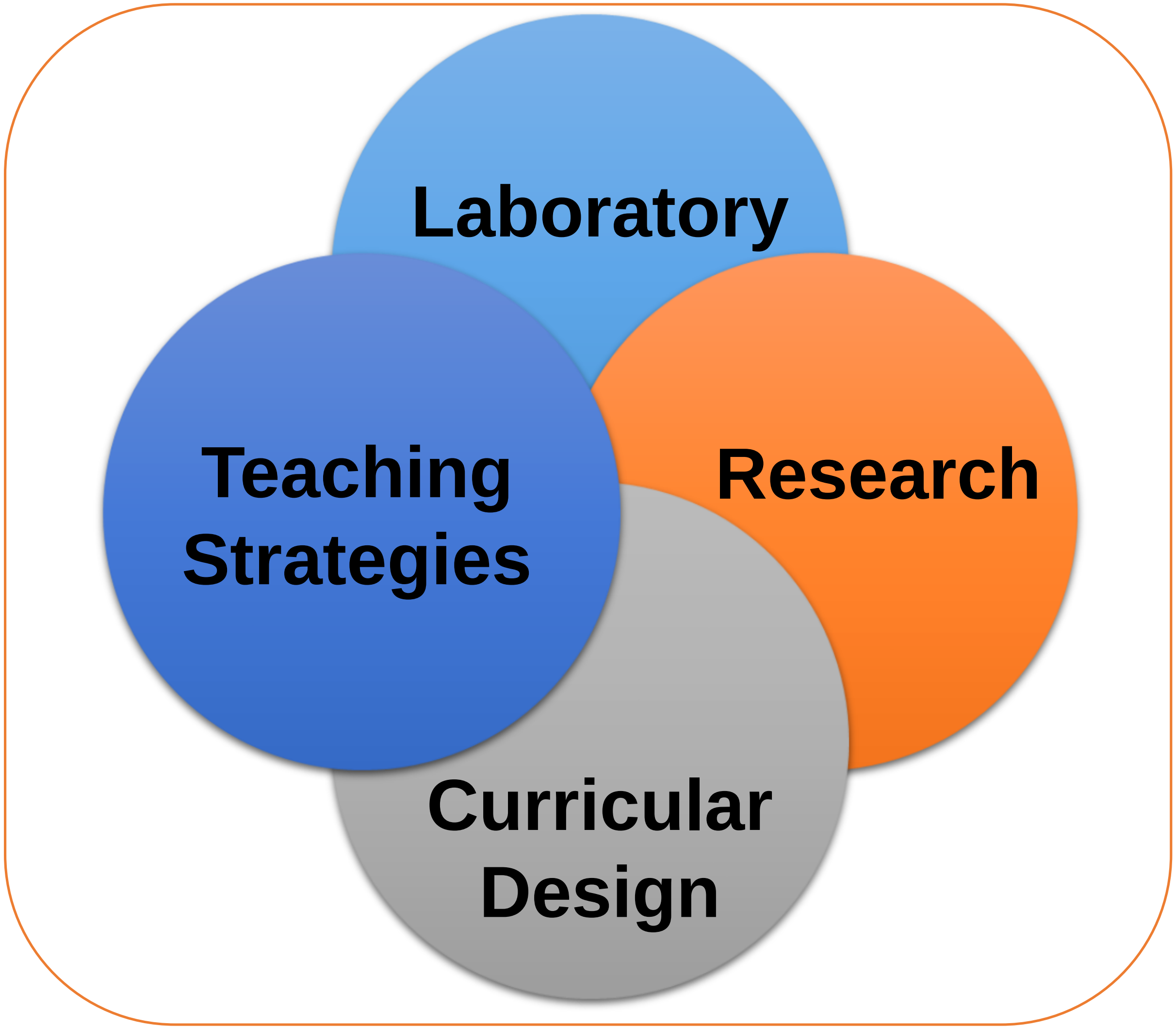
- Vectors
- Motion at constant speed
- Motion with constant acceleration
- Constant Acceleration quantitative
- Motion in Two Dimensions
- Energy
- Work
- Forces
- Forces of friction
- Momentum
- Forces of spring and circular motion
- Rotational and Harmonic Motion

Calculus content

- Linear model
- Quadratic model
- Derivatives
- Euler's method
- Non-continuous functions
- Integral
- Line integral
- Applications of derivatives and integrals
- Applications of mathematical models

Tecnológico de Monterrey

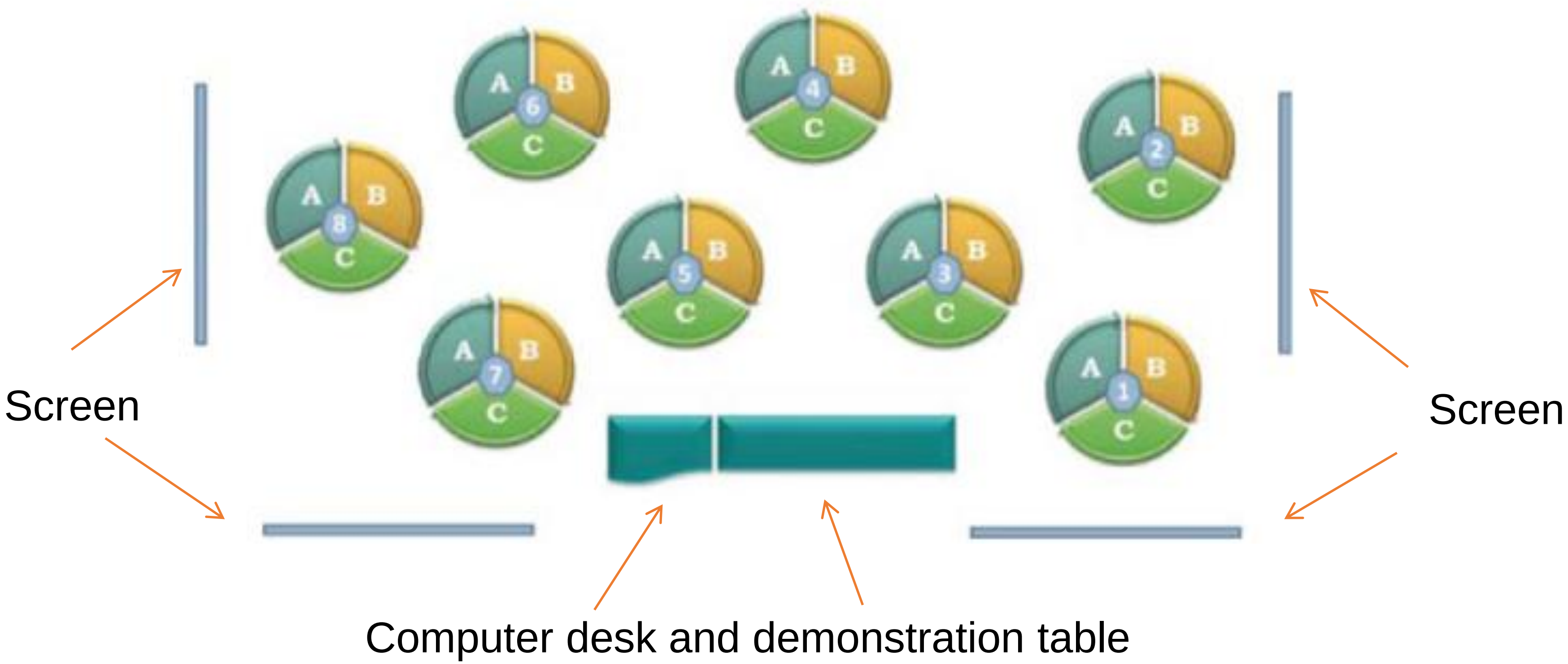
Classroom setting



Student-Centered Learning

Classroom setting

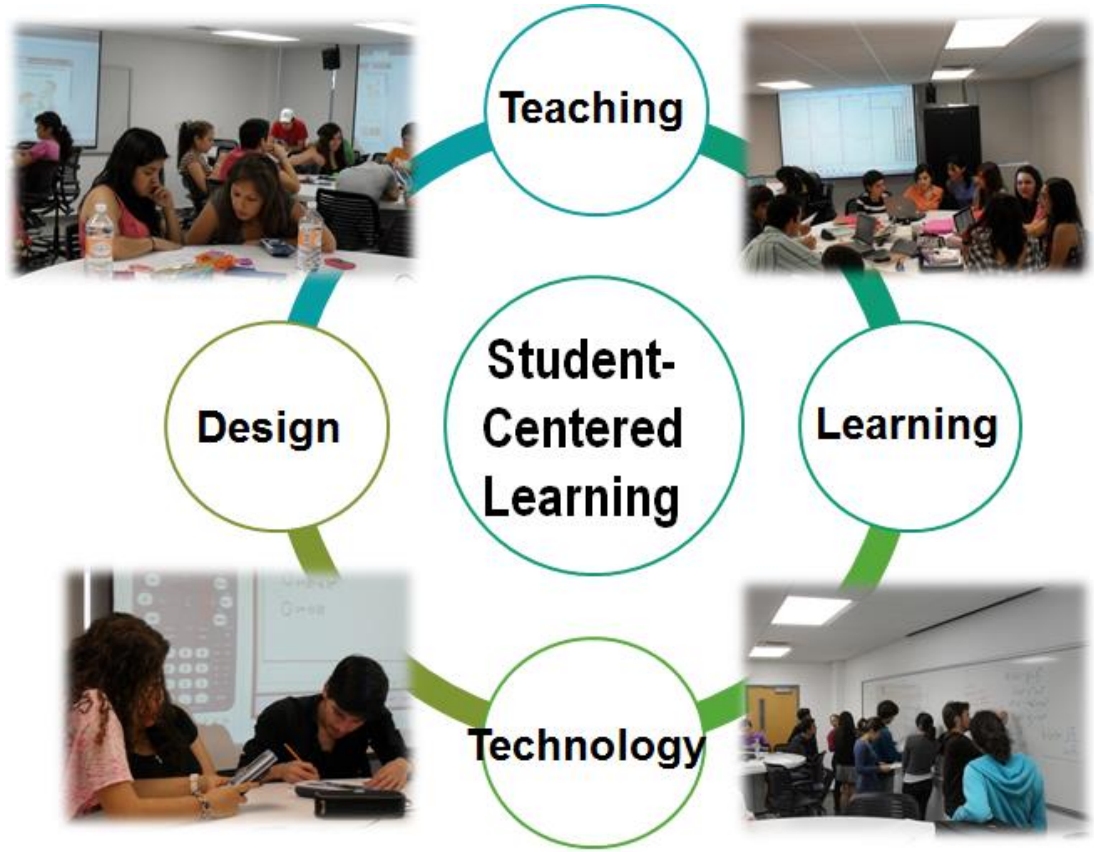
- SCALE-UP classroom type
  - ACE (Student-Centered Learning)



Tecnológico de Monterrey

Teaching Strategies

- Promote conceptual learning
- Modelling instruction
- Collaborative learning
- Modelling
- Using technology
  - Notebooks
  - Software
    - SimCalc MathWorlds, Excel, Graphing tools,
  - Laboratory Equipment
  - Internet

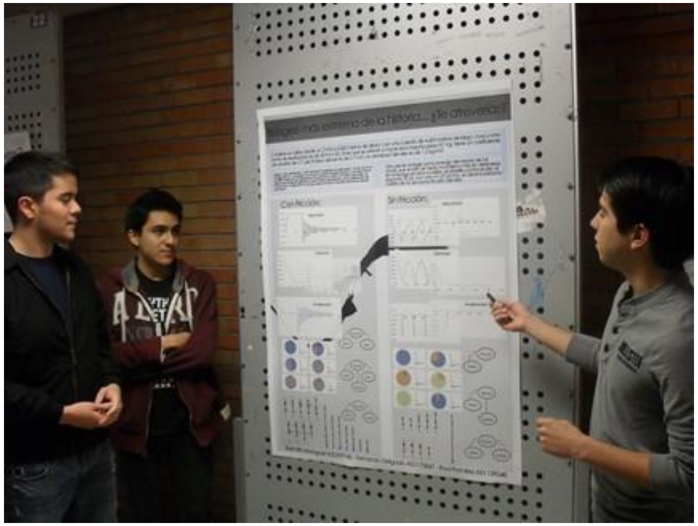


Use of technology and lab equipment

- Using technology
  - Notebooks
  - Graphing calculators
  - Software
    - SimCalc MathWorlds, Excel, Graphing tools,
  - Laboratory Equipment
  - Internet

Evaluation

- Align the pedagogy, content, classroom setting, and use of the technology
- Students present individual and collaborative integrated partial exams and work and present a collaborative final project.

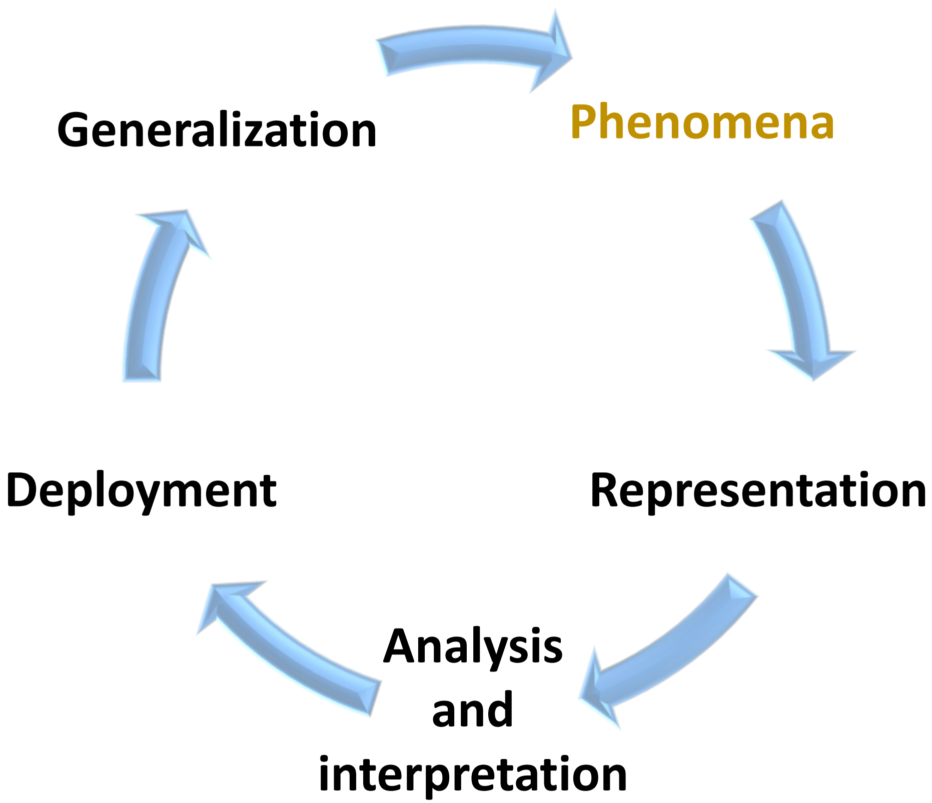


Faculty collaboration and engagement

- Teachers of Mathematics and Physics work together.
- All session are based on active learning.
- The course structure is based on Modelling Instruction
- For the final project is put into play several representations that they had to manage to sustain its solution.
- Students recognize that the course of Fis-Mat is a good learning experience.

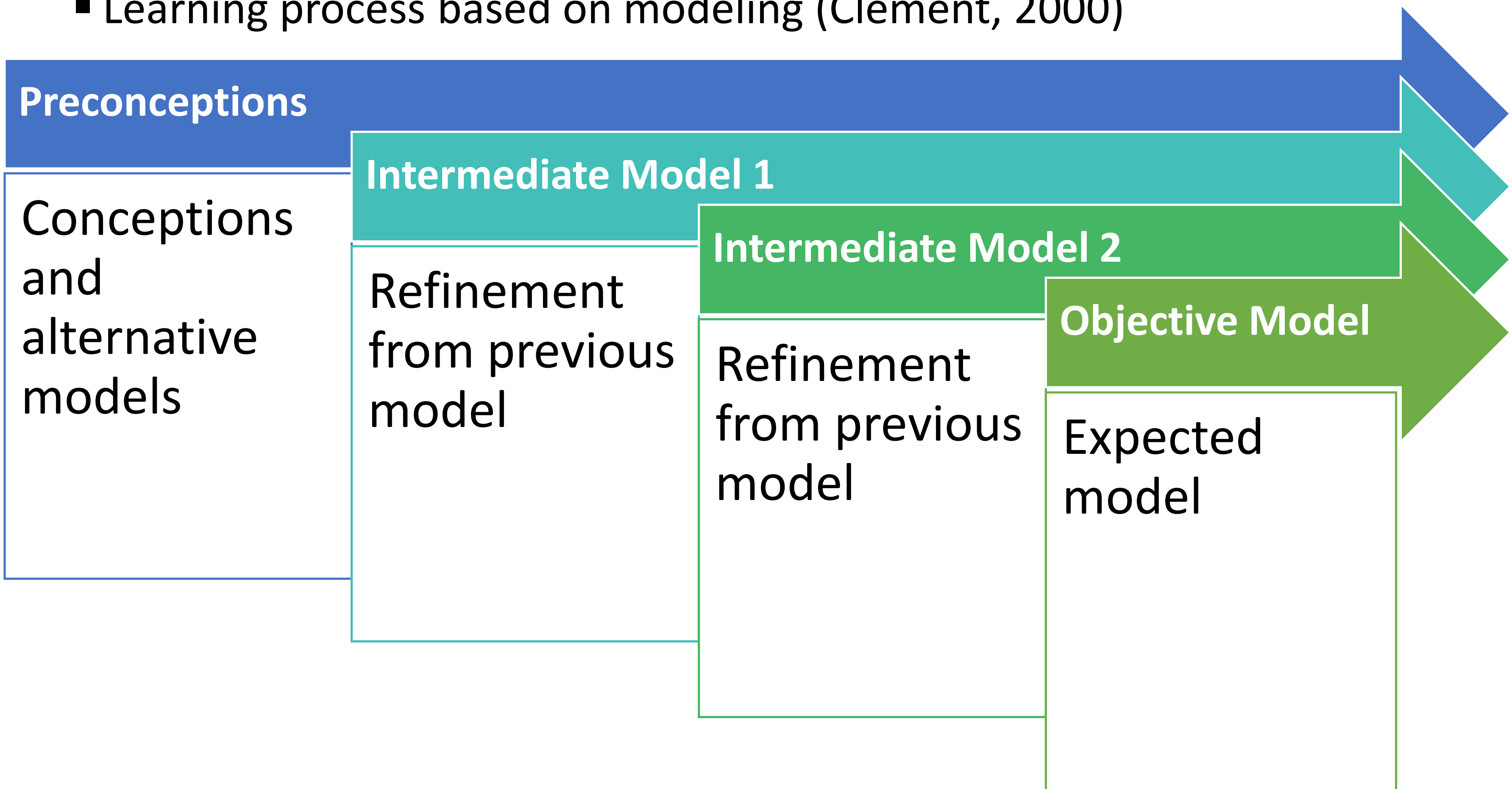
Modeling Instruction

- Students are actively engaged in a cycle of **model development**.
- Conveys content through student investigations of various models that are consistent with observations and notions (qualitative) and measurements (quantitative).
- A modeling course focuses on model development and testing in the context of physics.



Modeling Instruction

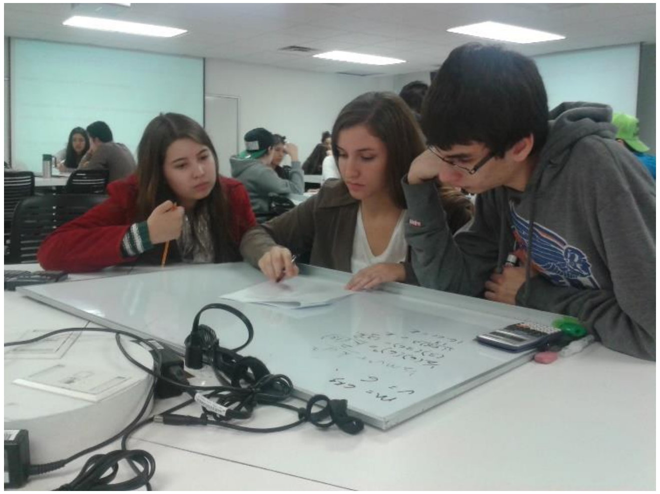
- Learning process based on modeling (Clement, 2000)



Halloun, I. A. (2004). *Modeling Theory in Science Education* .Springer .

Modeling Instruction

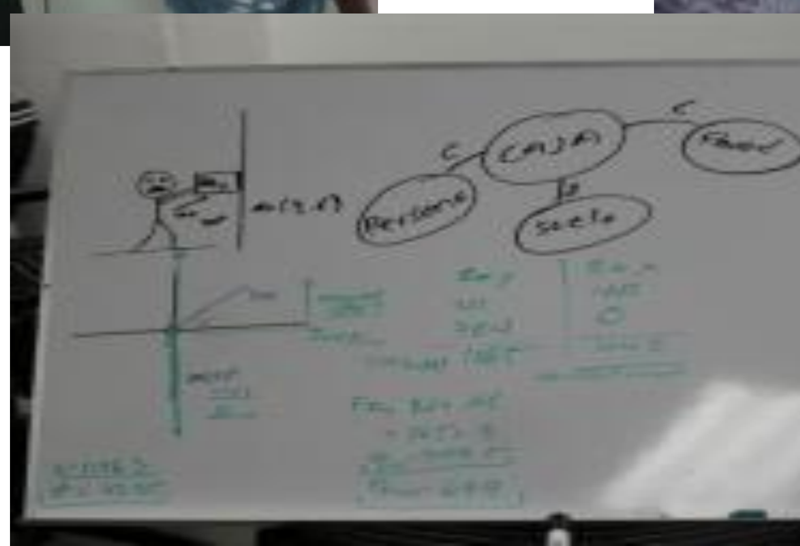
Work in collaborative groups → Report solution on portable whiteboards → Discuss in "big circle" format (all students)



Modeling Instruction

- Formal collaborative groups (groups of 3)
- Report solution on portable whiteboards
- Big circle discussion (all students)

Investigating, presenting and discussing



Academic Achievement

Academic achievement in Physics

- Final departmental evaluation
  - Includes entire curriculum of the course
  - 20 multiple choice items:
    - 10 conceptual and 10 procedural questions
  - Four version applied to all Physics sections at the same time
  - Automatic optical grading system

Academic achievement in Physics

- *Mean and standard deviation of the final test scores for three different groups*

	Honors/Physics Majors	Fis-Mat	Other groups
Mean	77.2	65.5	59.8
Std Dev	17.2	18.8	19.9
Number	120	20	595

Academic achievement in Physics

- Students in the experimental group scored higher than those in the rest of the groups; however, they scored less than those in the honors/physics majors groups.

✓**Encouraging results considering this is the first experience**

Academic achievement in Mathematics

- No final departmental evaluation
- Comparison was among groups from other semesters, same course, same teacher
- Same mathematical curriculum
- Same final exam problems
- Same grading schema

Academic achievement in Mathematics

- *Percentages of correct response on the final test problems for five different math groups*

	Fis-Mat (20)	A (40)	B (40)	C (39)	D (40)
Prob 1	**89%**	74%	62%	80%	76%
Prob2	**96%**	82%	81%	80%	81%
Prob3	84%	85%	85%	**86%**	76%
Prob4	**89%**	83%	73%	84%	75%
Prob5	**98%**	88%	90%		
Prob6	**63%**	53%	59%		
Prob7	**88%**	76%	65%	74%	69%
Prob8	**93%**	63%	59%	66%	58%

Tecnológico de Monterrey

Academic achievement in Mathematics

- Students in the Fis-Mat course were able to respond to the same exam and in many problems do as well or better than those students in a regular calculus course.

✓ **Encouraging results considering this is the first experience**

Models and Representations

Representations

- The representations are the pieces used to construct **models**. For example:

Spring-Mass System Project

On a table there is a block attached to a spring, and the spring is attached to a wall. A person slides the block onto the table, stretching the spring. Then, the block is released (still attached to the spring) and it slides onto the table.

Use the modelling tools to analyse the situation and complete the following tasks:

1. Determine the friction coefficient between the block and the table.
2. Determine the spring constant.
3. Construct a mathematical model that describes the motion of the object at any time from the moment the block is released.

Representations

- This course we fosters:
  - **Drawing of the situation**
  - **Assumptions**
  - System Schema
  - Motion Map
  - Energy pie charts
  - Force diagrams
  - Graphs
  - Mathematical repres

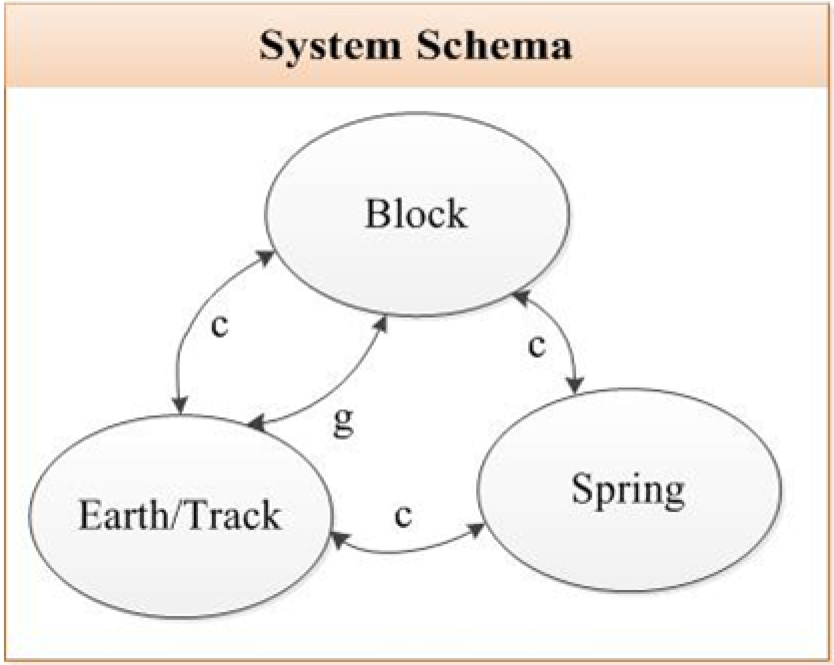
Physical situation

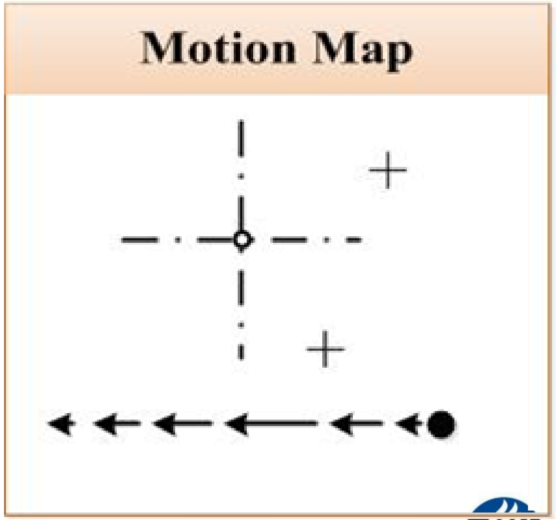
Assumptions

- Not a constant acceleration model
- Spring has no mass
- Spring starts stretched
- Model applies until spring has reached natural length (uncompressible spring)
- Track has friction

Representations

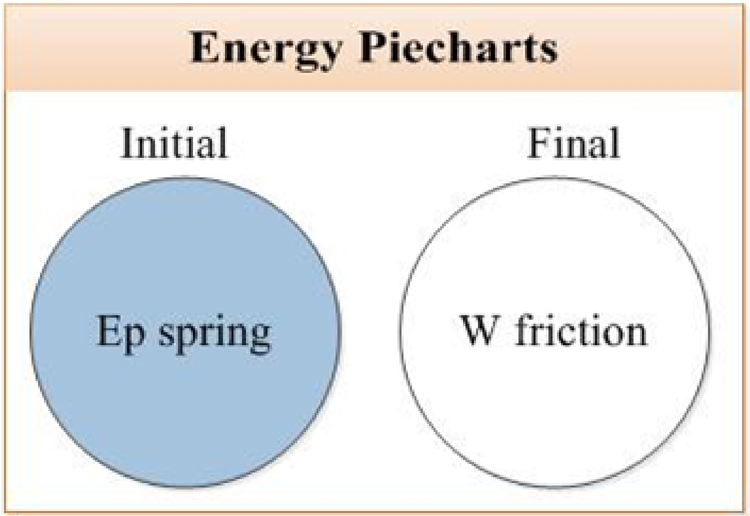
- This course we fosters:
  - Drawing of the situation
  - Assumptions
  - **System Schema**
  - **Motion Map**
  - Energy pie charts
  - Force diagrams
  - Graphs
  - Mathematical representation

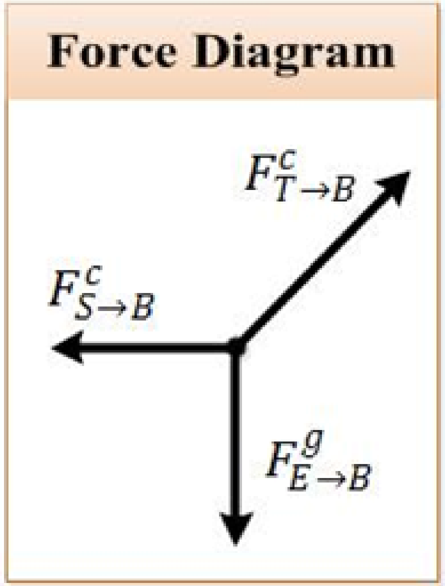




Representations

- This course we fosters:
  - Drawing of the situation
  - Assumptions
  - System Schema
  - Motion Map
  - **Energy pie charts**
  - **Force diagrams**
  - Graphs
  - Mathematical representation





Tecnológico de Monterrey

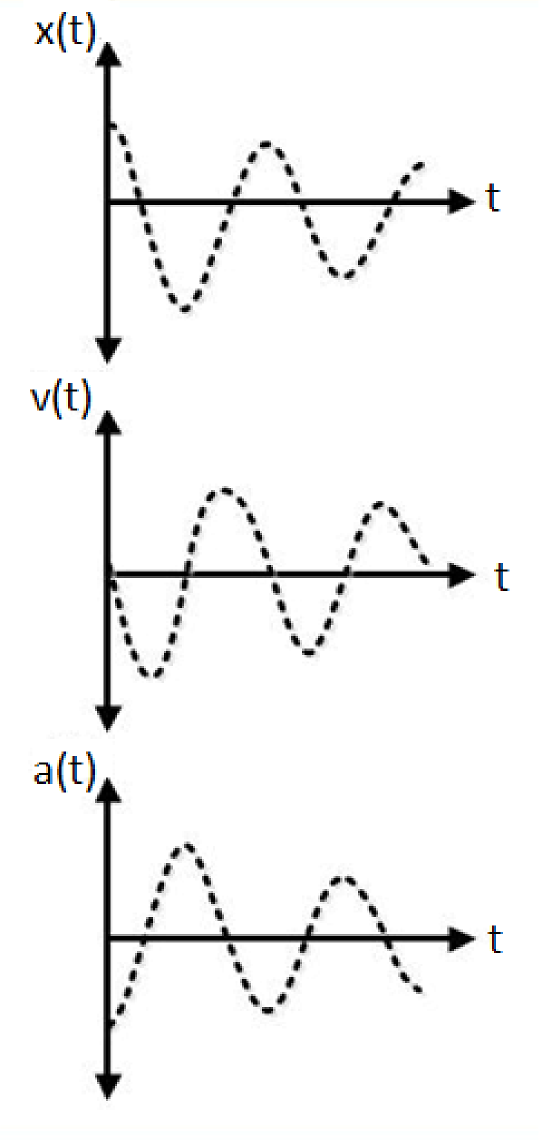
Representations

- This course we fosters:
  - Drawing of the situation
  - Assumptions
  - System Schema
  - Motion Map
  - Energy pie chart
  - Force diagrams
  - **Graphs**
  - **Mathematical representation**

Acceleration model

$$a = \frac{-k\Delta x + \mu_k mg}{m}$$

Graphs

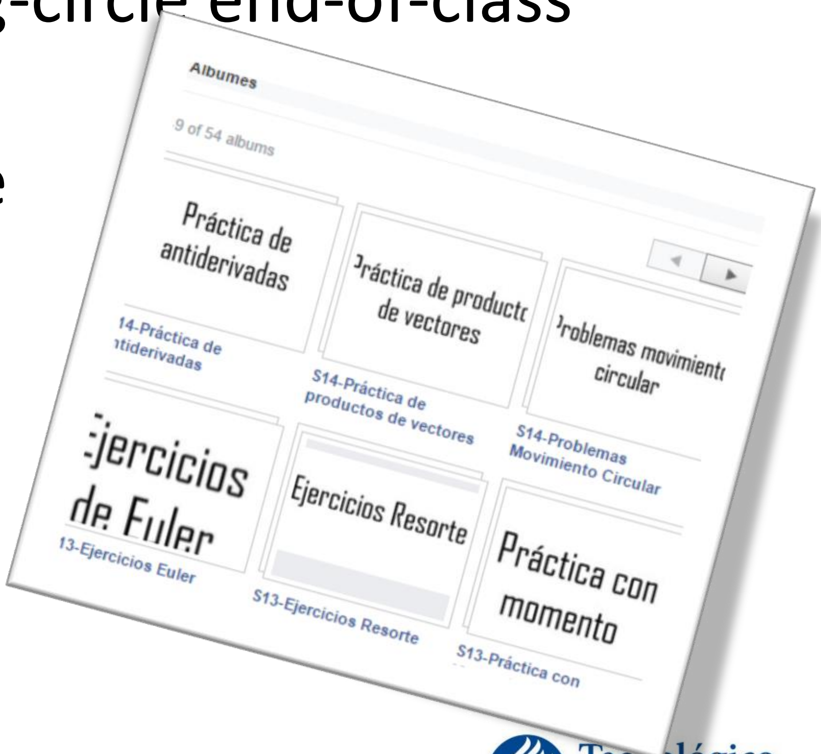


Research Questions

- How do **students' models** change overtime in an integrated physics and mathematics course?
- How do **students' representation** change overtime in an integrated physics and mathematics course?

Class Work

- Group collaboration activities are worked in portable whiteboards.
- Small groups work allowed big-circle end-of-class discussion.
- Pictures of all whiteboards are shared and organized in a Facebook Group.
- The whiteboard pictures served as notes for students.



Tecnológico de Monterrey

Analysis of students work

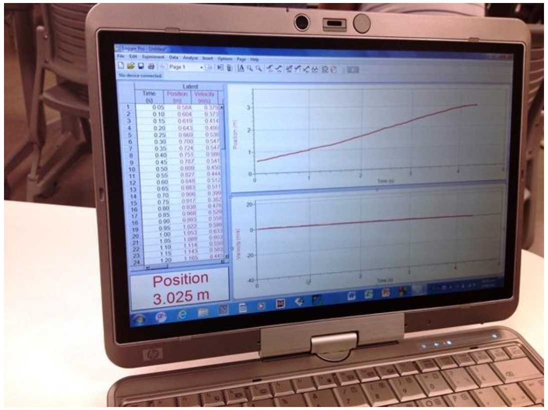
- Students work is analysed in three moments:
  - Beginning of the semester (Week 1).
  - Middle of the semester (Weeks 5 to 7).
  - End of the semester (Week 11).

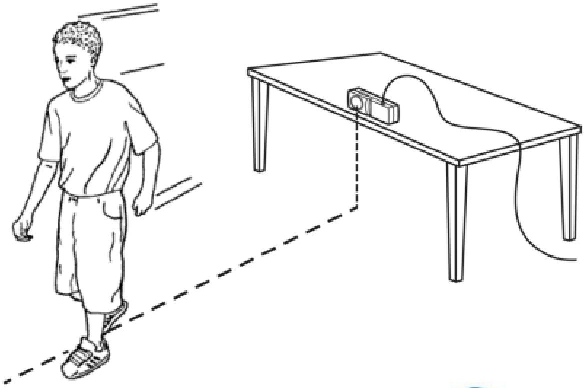
Analysis: Beginning of the semester

- Students responses focused on providing a **concrete answer** to the exercise. They give less value to the process in solving the problem or to provide conclusions or interpretation to the numerical answer.
- Also, students tried to reach an answer through a **single procedure**. They were not aware that there could be different ways to reach a solution or even different possible answers.

Week 1: Investigating constant velocity

- In each of the next four trials, you have to try to move in a way such that you match the given representation and fill in the other two representations. (i.e. you are given a velocity vs. time graph and have to move to create the same velocity vs. time graph)

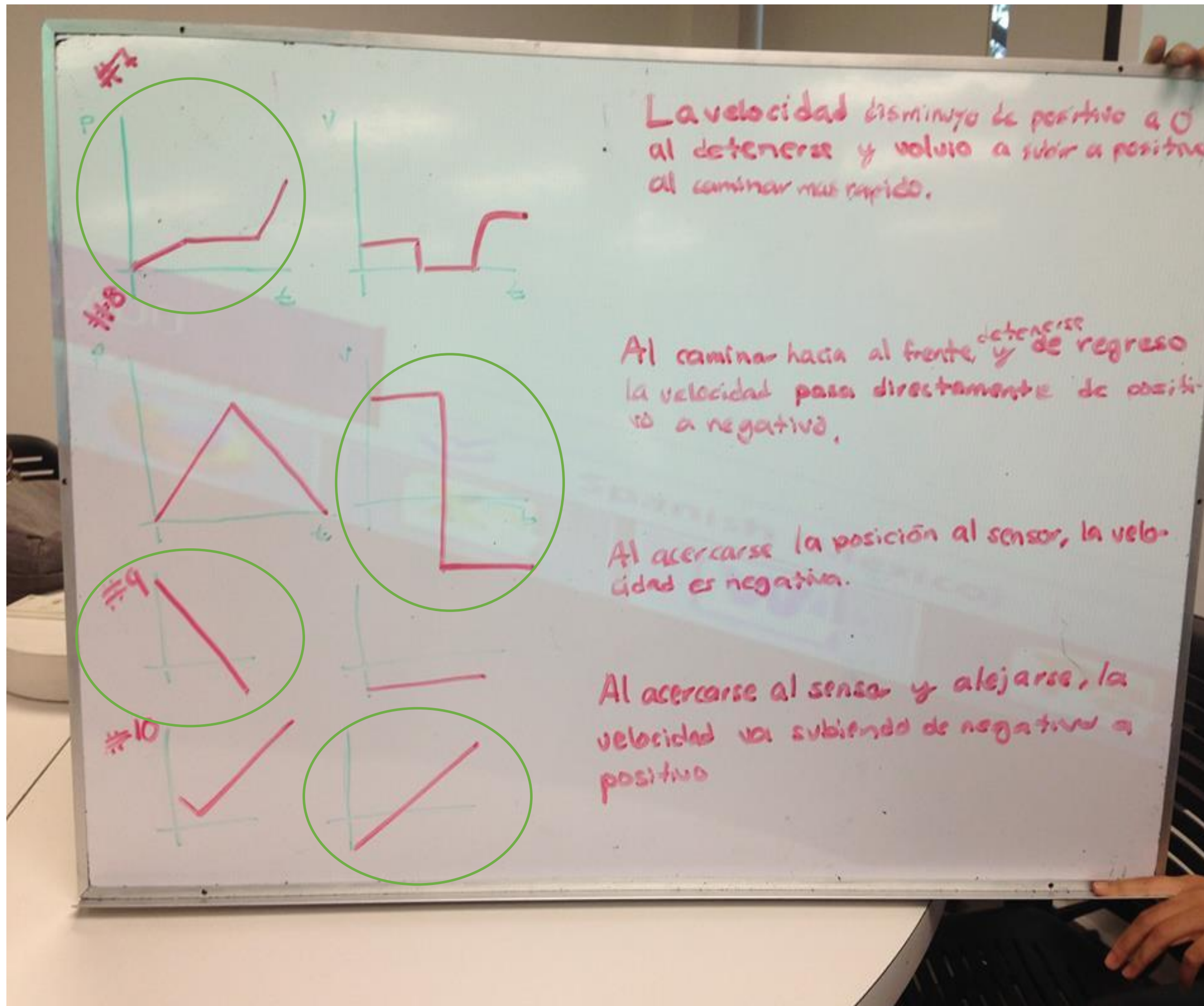




Tecnológico de Monterrey

Week 1: Investigating constant velocity

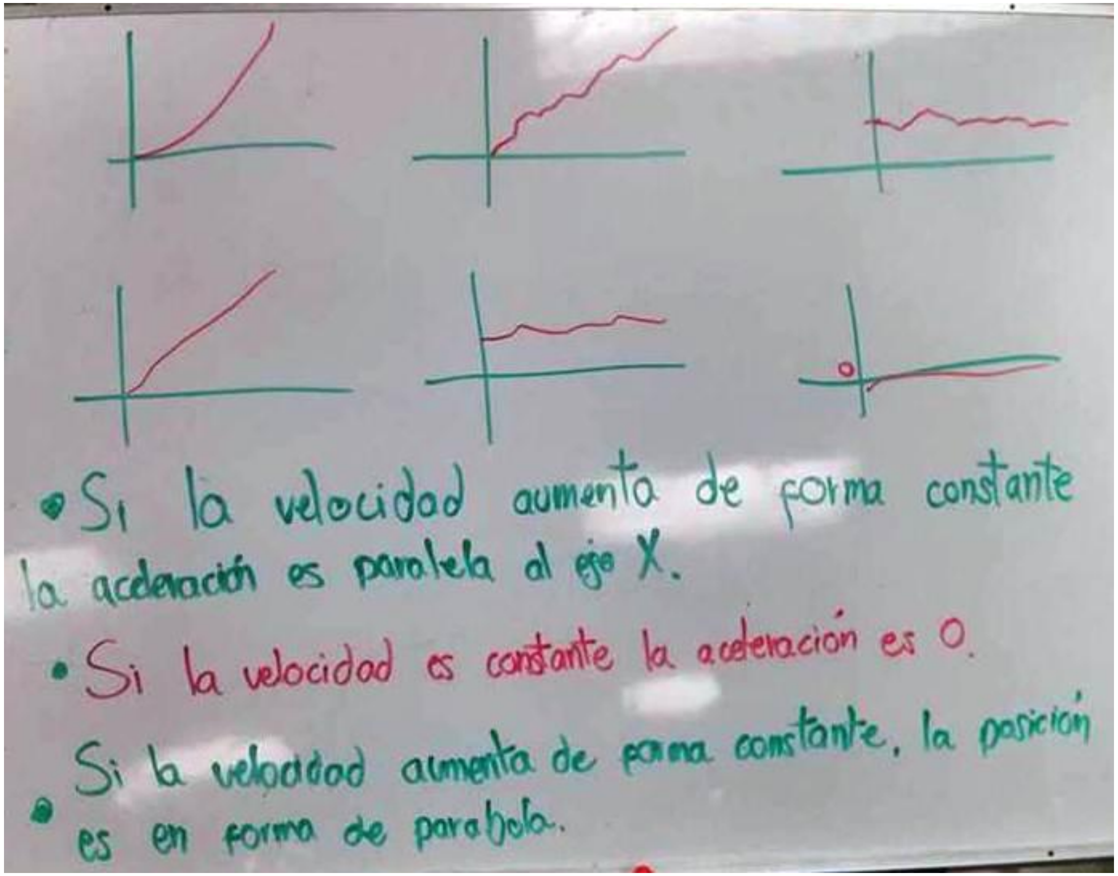
- Work done to the dot. No making sense was involved in completing the exercise.



Given graph

Week 1: Investigating constant velocity

- Simple drawings.
- Not well defined graphs.
- Focusing in the instruction not on the making sense.
- Axis not labeled in the graphs
- Graphs were copied literally from the computer.

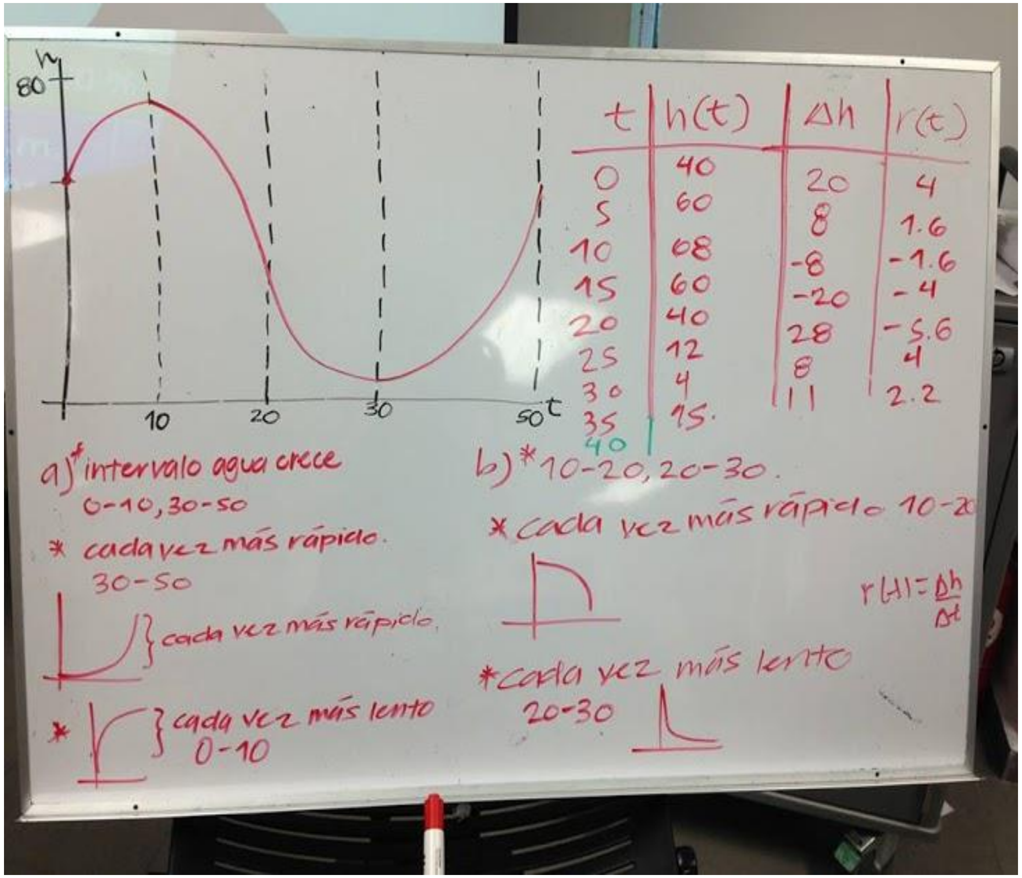


Analysis: Middle of the semester

- Some teams began to connect the mathematical elements to the physical context.
- More structured solutions. Richer discussions evidence how the model construction development in solving the activities.
- Frequent use of three different representations to validate their solution: algorithmic procedures, graphical representations and argumentation are congruent (big circle discussions).

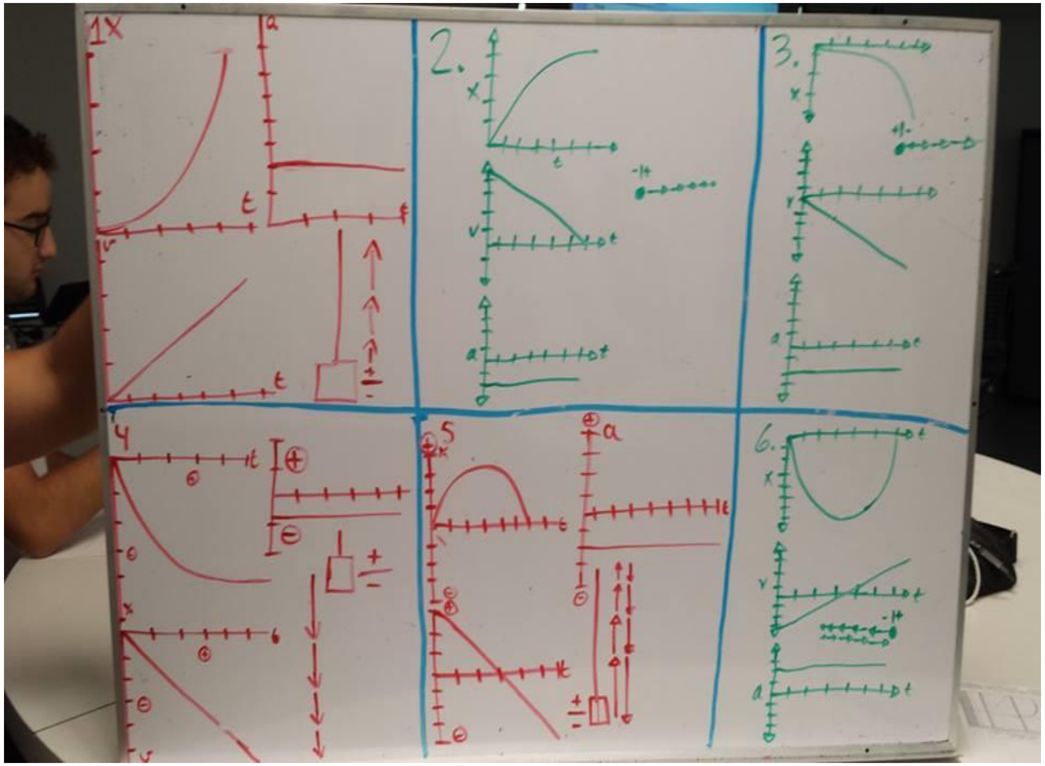
Week 5: Qualitative analysis

- Students use graphical and numerical representations to outline the solution in the whiteboards.



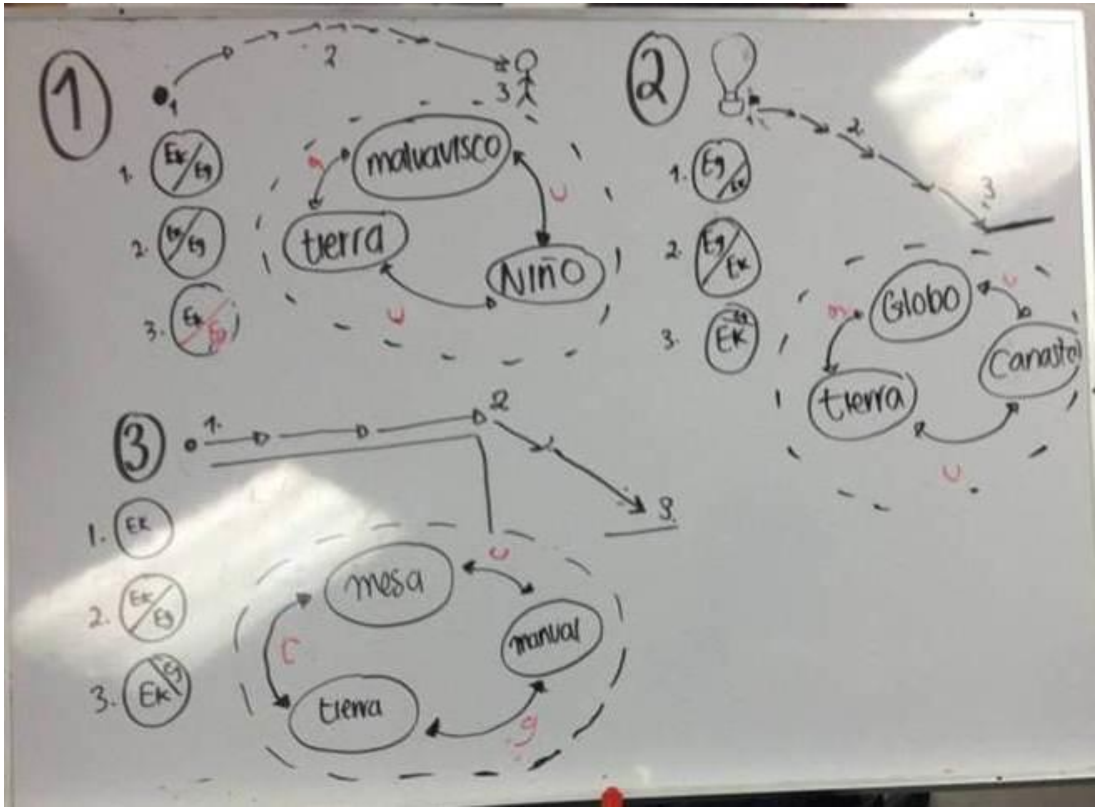
Week 5: Qualitative analysis

- Connectivity of mathematics tools and physics tools to solve the activities.



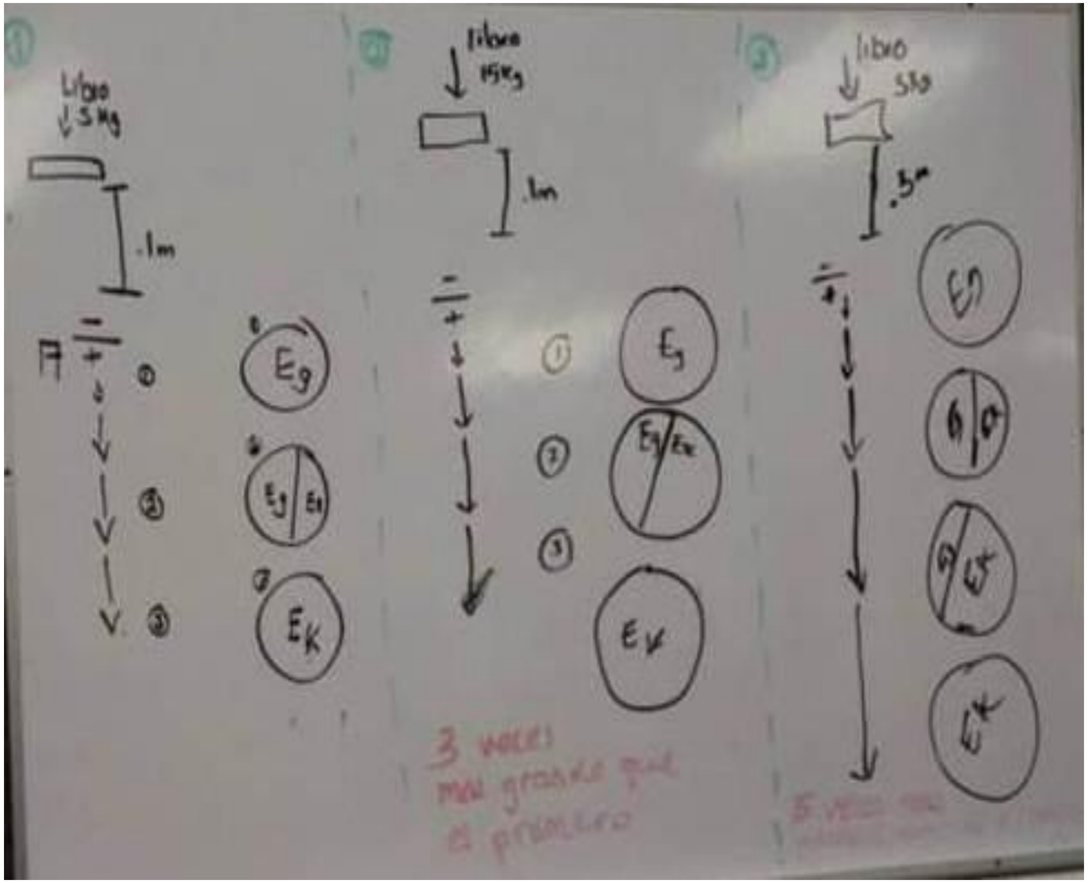
Week 6: Practice with energy pie charts

- Students use more drawings and planning into the whiteboards.



Week 7: Investigating energy conservation

- Variety of representations.
- Better organized ideas on the whiteboard.

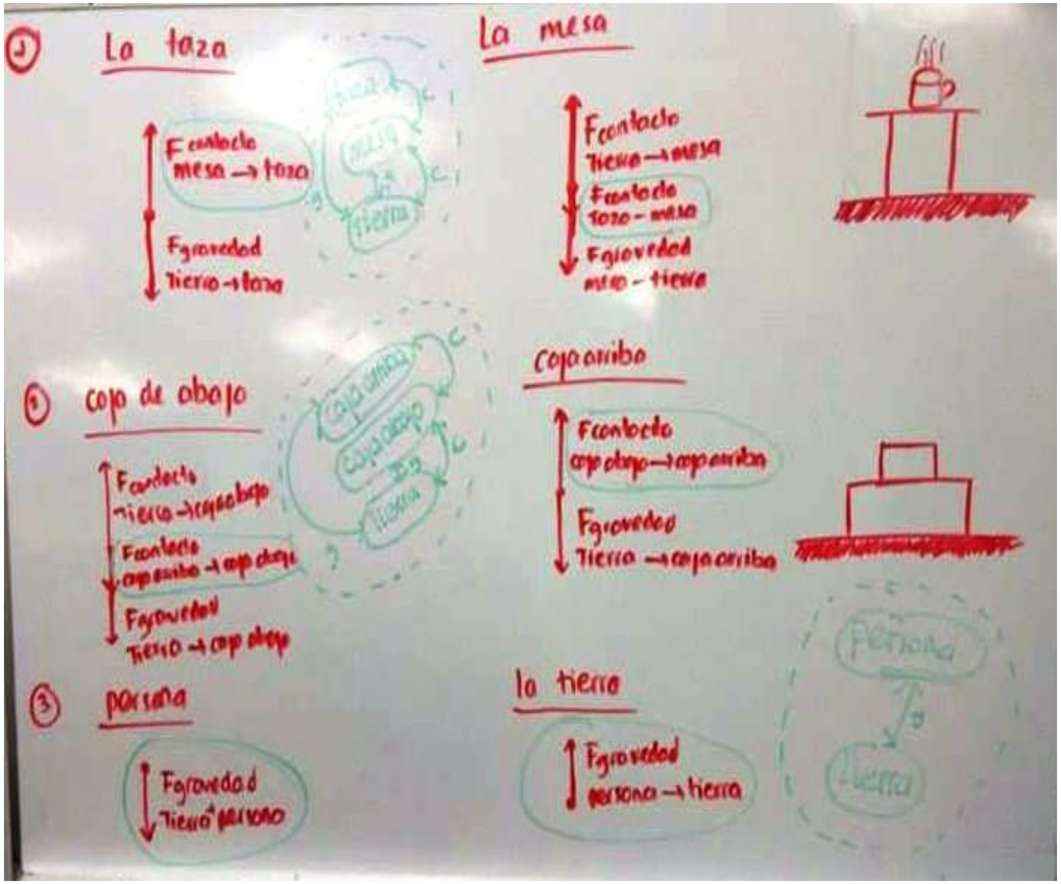


Analysis: End of the semester

- For the final project is put into play several representations that they had to manage to sustain its solution.
- They rely on the mathematical elements such as the use of a numerical approximation, the graphic and algebraic part to help them build the solution of the exercise.
- Also, different types of representations that were active during the semester in physical contexts, such as: diagrams, graphs analyzing and using the necessary equations in the solution of the exercise.

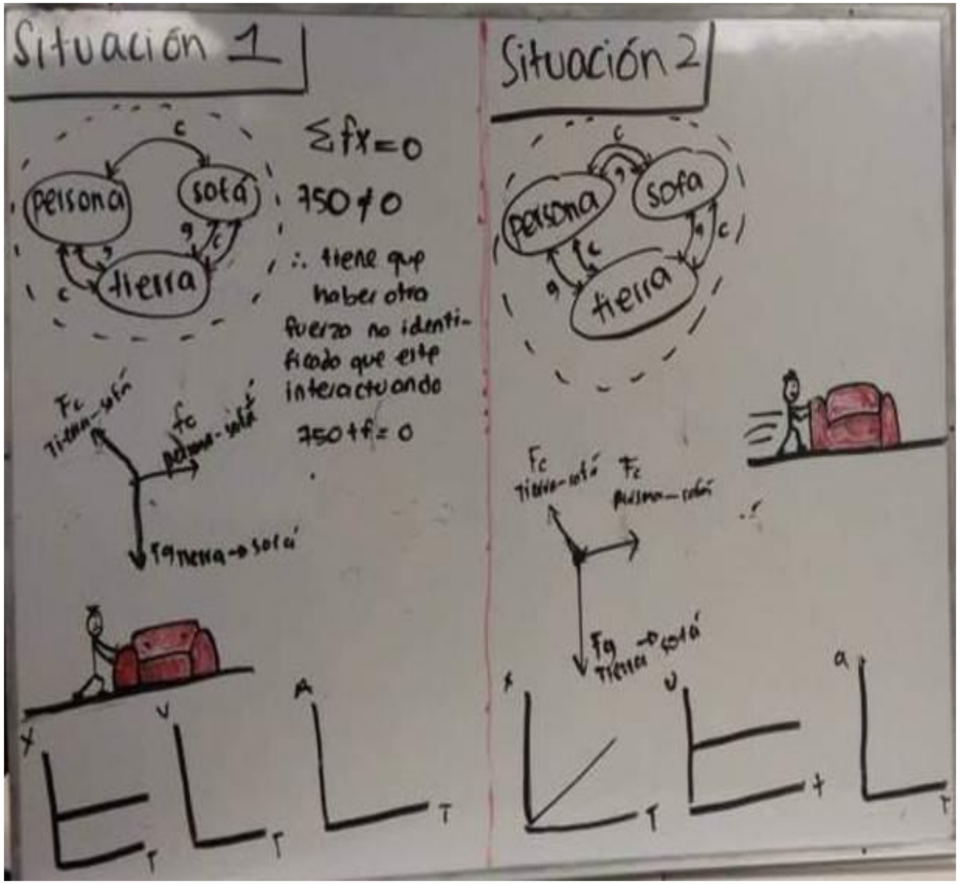
Week 11: Practice with 3rd Newton's Law

- Whiteboards seem ordered and planned.
- Drawings added for easier explanation of the specific situation.



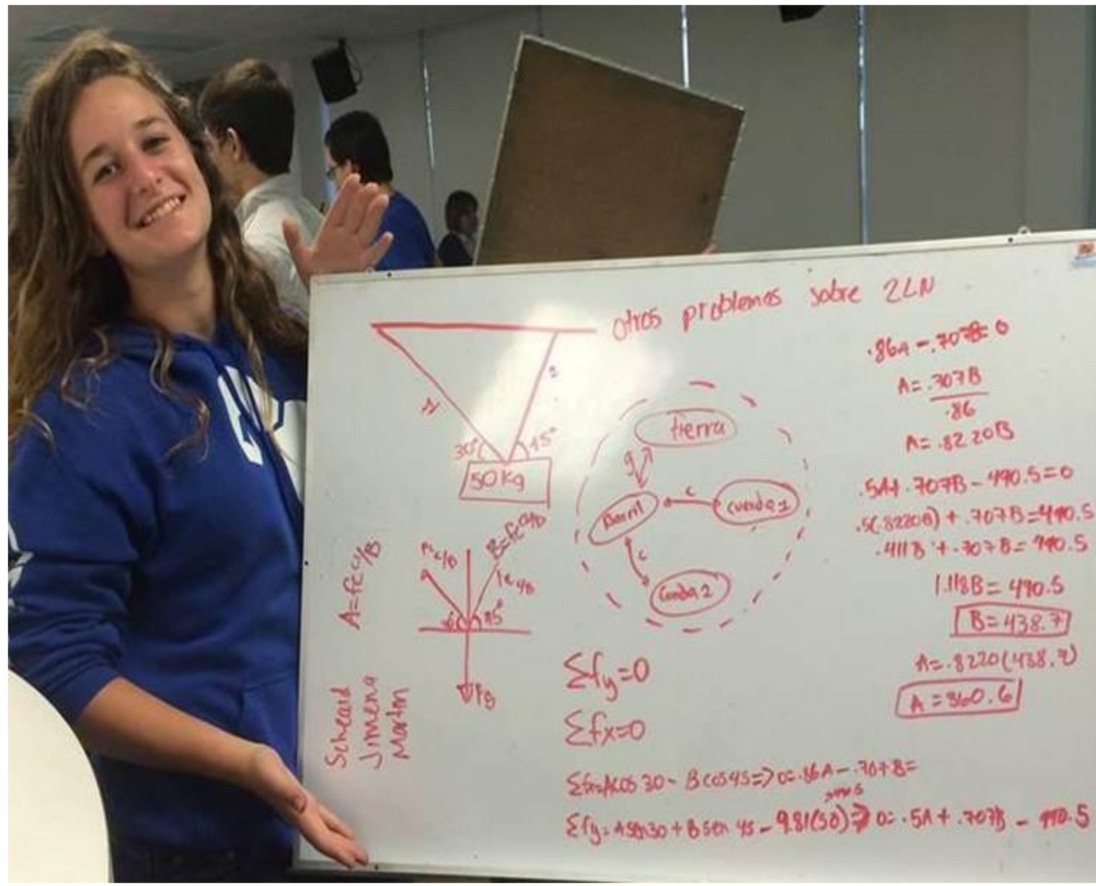
Week 11: Investigating frictional forces

- Use of drawing to represent the phenomenon under study. Students used graphs, words and algebraic representations the indicate the behavior of the velocity.



Week 11: Hanging block problem

- Whiteboards are easier to share at this point, not only due to clarity, but also since everyone has the same base knowledge of the models.



How do students' models change overtime?

- There is evidence of models evolving into more robust models.
- At beginning of the semester, students used a single model and looked for a concrete answer.
- Later in the semester, students valued constructing a more robust model.
- Moreover, students start using the tools learned to solve problems in general (physics oriented of math oriented problems).
- Students commented (in informal conversations) that this course encourages to learn and to enjoy the process of learning.

How do students' representations change overtime?

- There is evidence of an increase on the variety of the representations as well as the congruence among them.
- At beginning of the semester, students used procedures that they recall from memory rather than from reasoning.
- Later in the semester, students valued to work together, bounced ideas with peers, summarized the main ideas that emerged from the activity and used a variety of representations.

How do students' representations change overtime?

- Integrating physics and mathematics content allowed students to recognize the usefulness of the mathematical tools to solve a problem (within a context, applied problem).
- Students value having different representations and were more aware of the congruence among the different representations as it help the to pay more attention to details.
- It was observed that students were more at ease moving form one representation to another.

Model-Application Activity

Participants

- 54 first year engineering students worked in groups = 18 groups of three students
- Same collaborative groups since the beginning of the semester
- Quadratic model started in the second week into the semester and ended by the fourth week with the ball in a cup activity

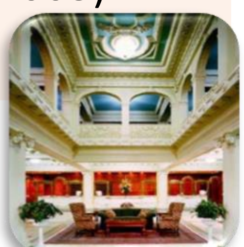
Our Course: Starting Week 3

Modeling Instruction

Week 2
Session 1

Model-Eliciting Activity (MEA)

- Historic Hotel (Aliprantis & Carmona, 2003)



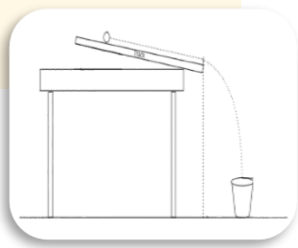
Model-Exploration Activity (MXA)

- Multiple exploration activities

Week 4
Session 3

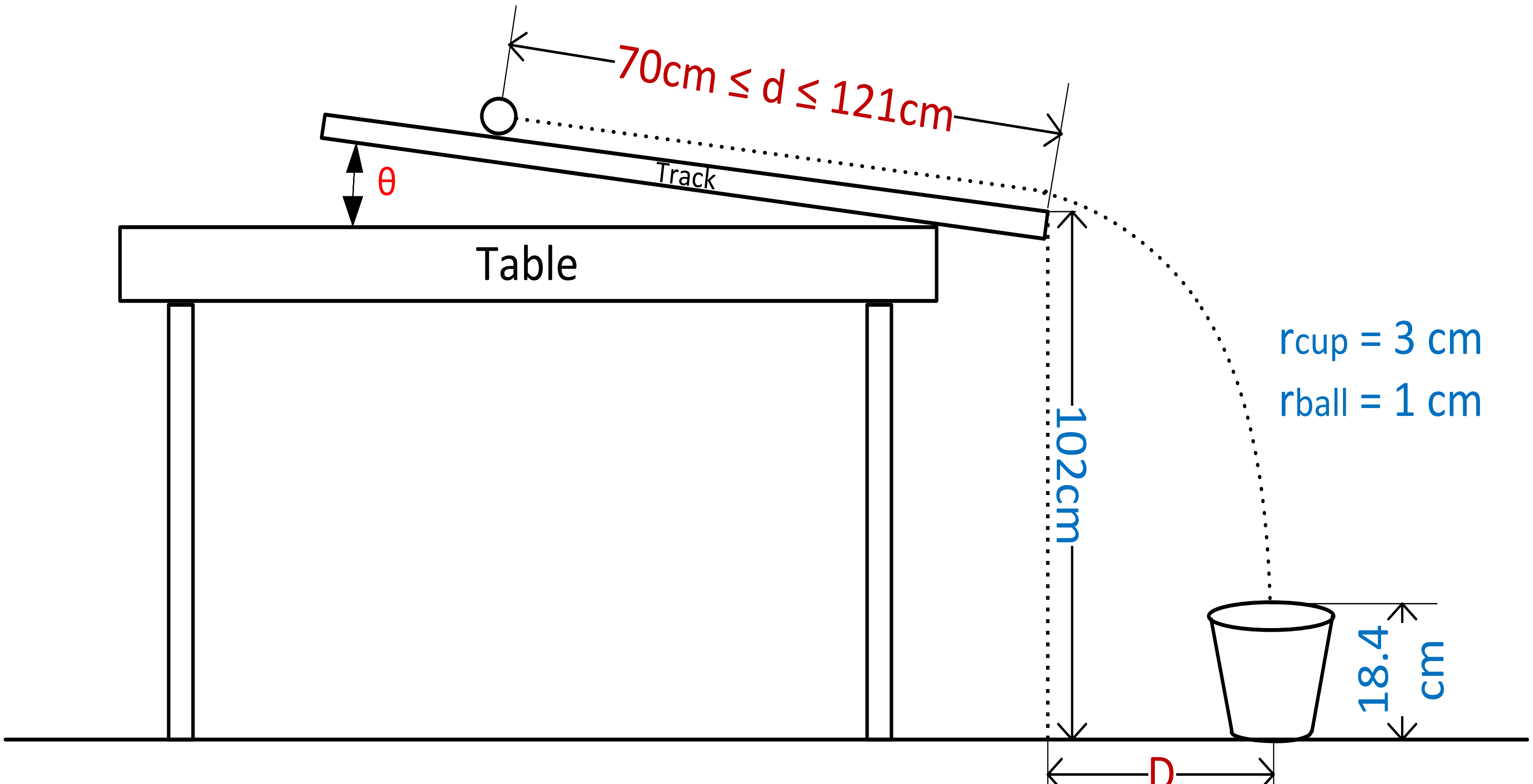
Model-Application Activity (MAA)

- Rolling ball in a ramp that falls into a cup



Model Application Activity

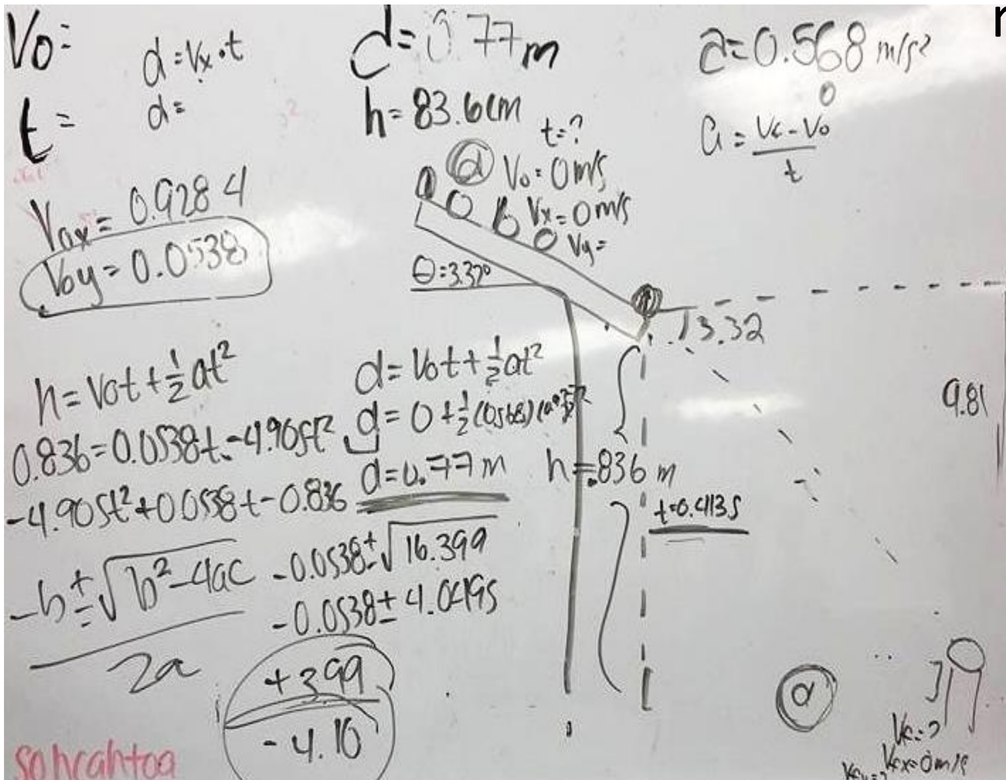
Determine where to put a cup (D) so that a ball let go from a ramp (d) falls into it.

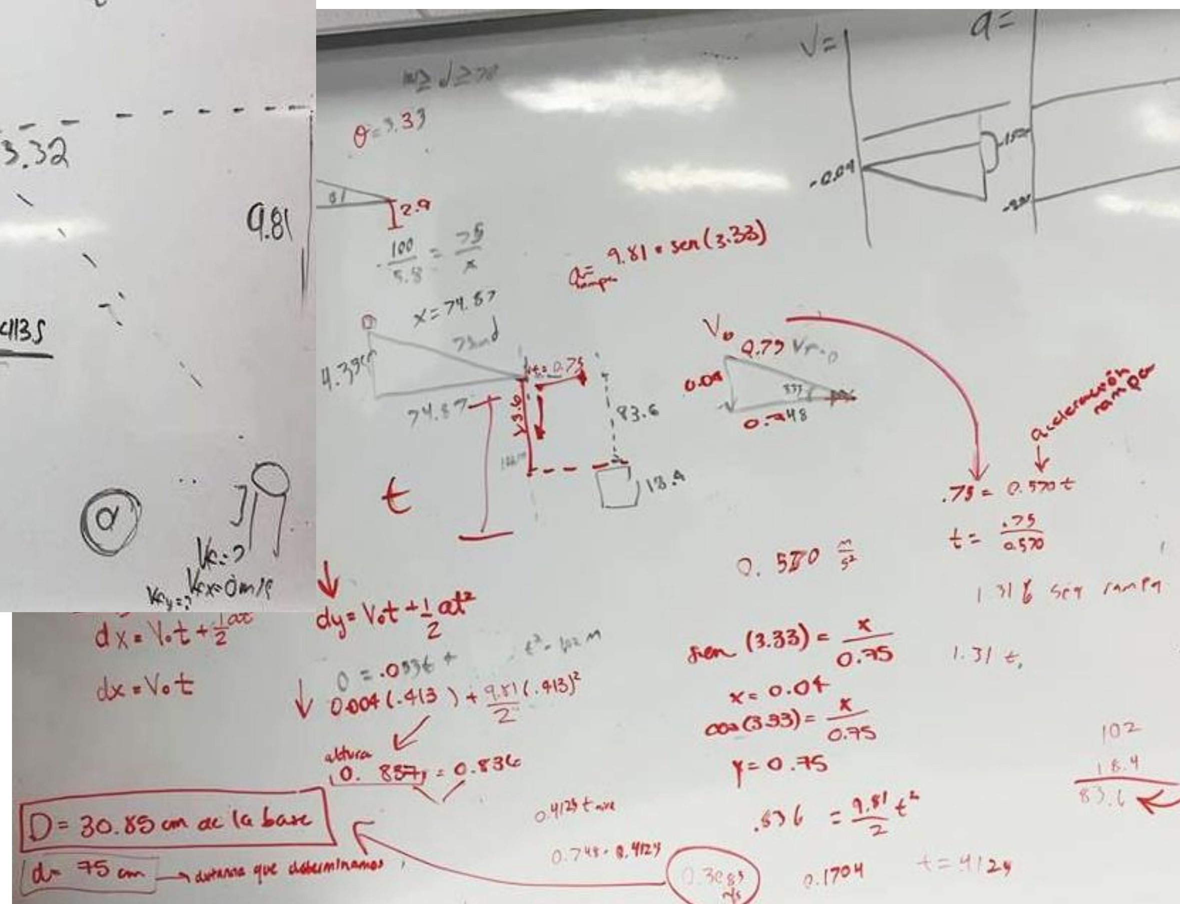


- Students work for 40 mintutes

Students work

Each of the 18 teams whiteboard was analyzed looking for which representations were present.





Discussion and results

- Solving this activity fulfills two purposes:
  - To apply and integrate the knowledge built until that point on the semester concerning both the physical and the mathematical models.
  - To empower students by bridging the gap between models built from ideal situations and those used in solving problems from real life situations.
- Students also realize that sometimes a drawing or combining different representations can help them better understand specific situations.

- The attitude of the students at the end of the session finding, whether (or not) their calculations and model actually work, indicates that this is an engaging and challenging activity.
- For the teams that their model made the ball hit the edge of the cup get to realize the importance of significant figures in their calculations.

Model construction
Final project

Participants

- 37 freshman students from a new program
  (Innovation and Development Engineering Program aimed to prepare students to develop comprehensive sustainable solutions in emerging fields of engineering)

Final project

- **Description:**

  The most extreme bungee jump in history

  of mankind consists of a cord of length L with an elasticity constant k that is tied to a UFO located at a height h from the ground. Then, a being jumps from the UFO. Would you dare?

Final project

Objectives:

- In teams of three create a **full model** that explains what happens after the being jumps from the UFO until it reaches the point closest to the ground. Consider the following two scenarios:
  - There is no air friction.
  - There is air friction.

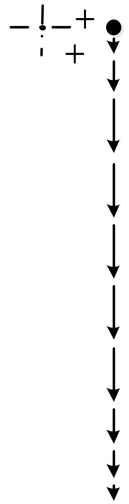
1. Establish the connections between physical and mathematical concepts and procedures.
2. Document their work, and prepare a poster presentation accompanied with an oral explanation of their reasoning.

Poster Fair

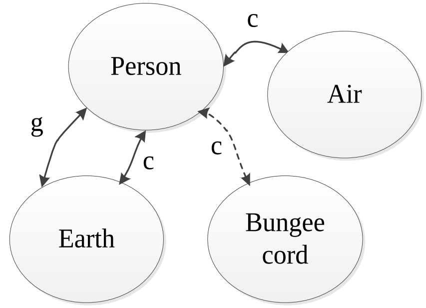
Complete model for the second part of the jump

Physical situation

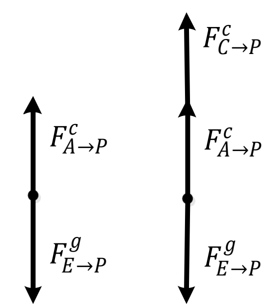
Motion Map



System Schema



Force Diagrams



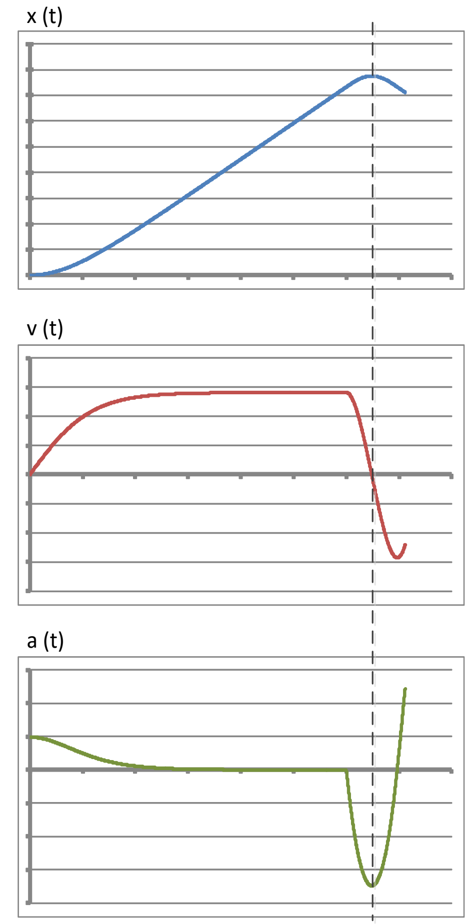
Assumptions

- Not a constant acceleration model
- Bungee cord behaves like a spring
- Bungee cord has no mass
- Two different sections, before and after the cord start stretching

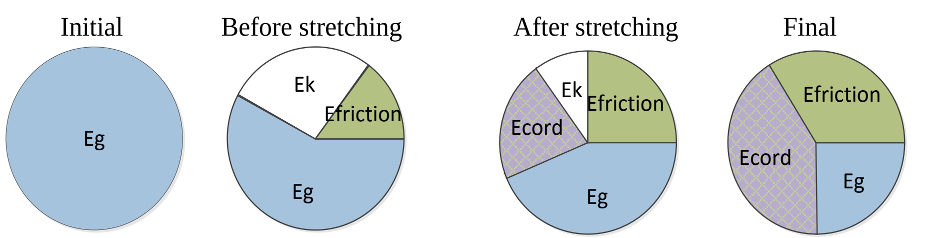
Newton's Second Law

$$\sum \vec{F} = m\vec{a}$$

Graphs



Energy Piecharts



Acceleration

$$a = \frac{g - 0.5C\rho A v^2}{m}$$

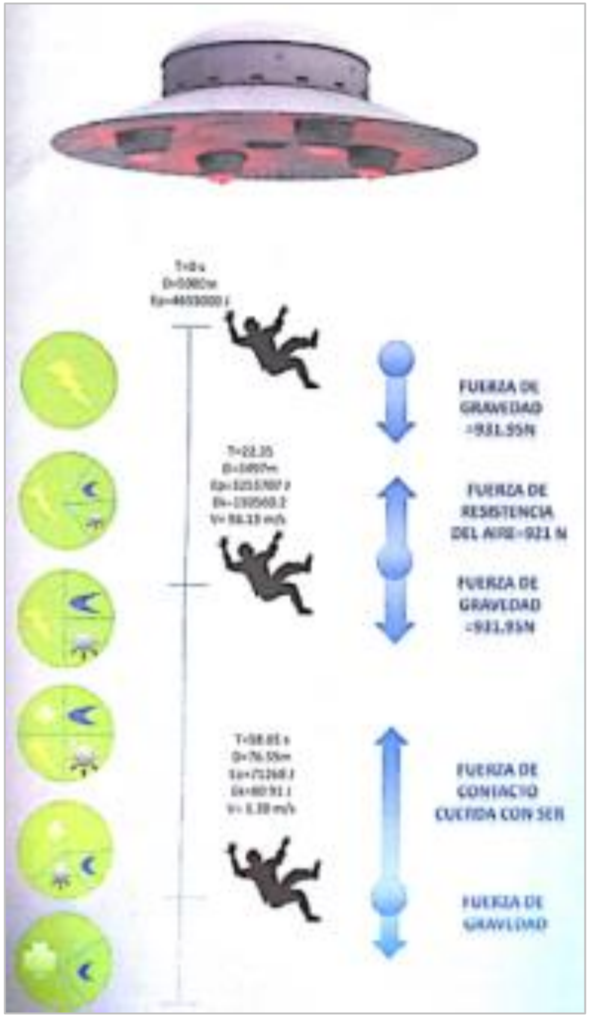
$$a = \frac{g - 0.5C\rho A v^2 - kd^2}{m}$$

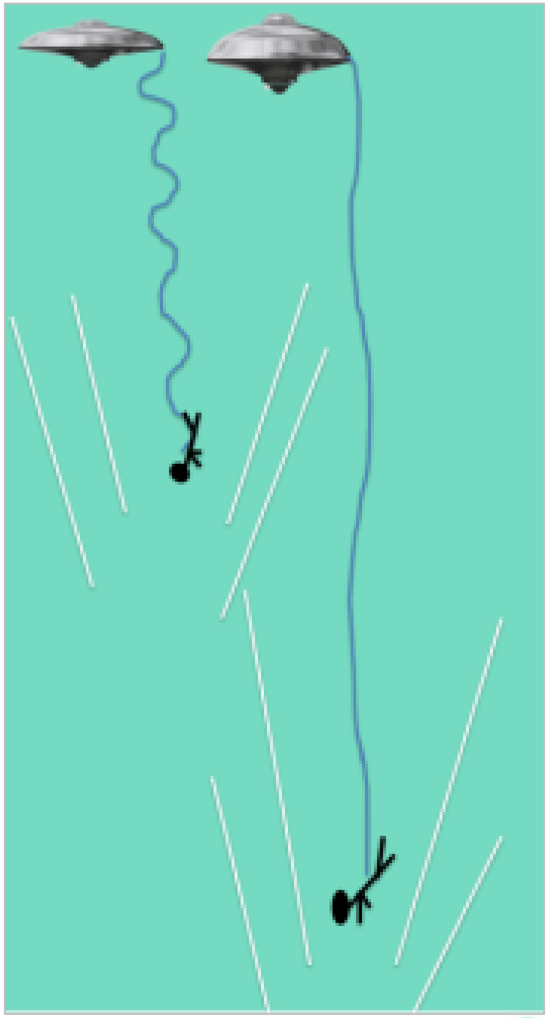
Energy conservation

$$E_{g1} = E_{g2} + W_{frict\,2} + E_{k2} = E_{g3} + W_{frict\,3} + E_{k3} + E_{cord\,3} = E_{g4} + W_{frict\,4} + E_{cord\,4}$$

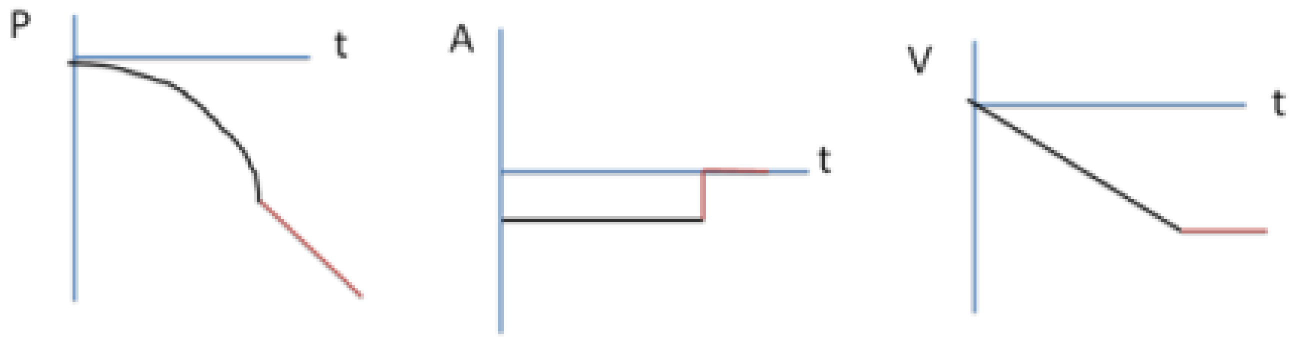
Tecnológico de Monterrey

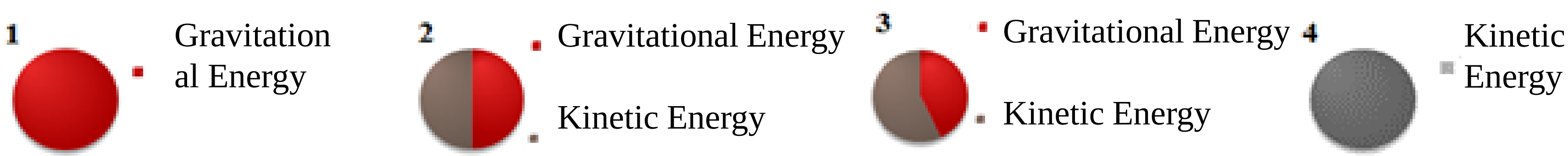
Comparison 1: Physical situation

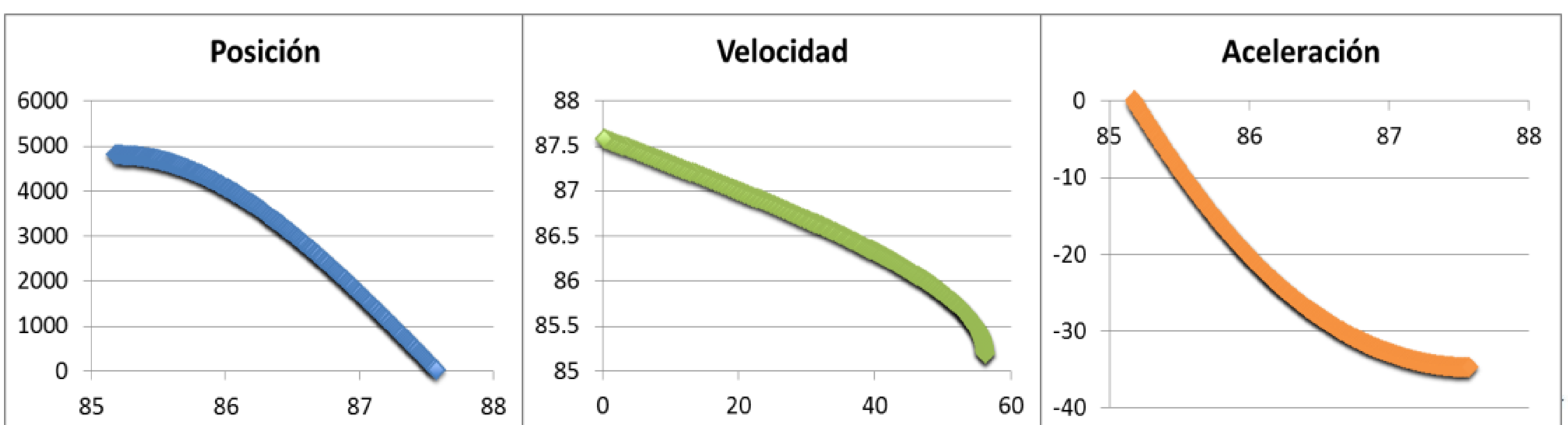




Example: Inconsistent motion map and graphs







Discussion and results

- 9 of the 12 teams arrived to a correct result in their Posters.
- All the teams that used Newton's Second Law and got an equation for acceleration arrived at the correct answer by using the Euler´s Method.
- Not getting an equation for acceleration makes students use the model corresponding to constant acceleration, which in a case with air friction is false.
- Finally, no team that explicitly used Euler's method got their answer wrong.

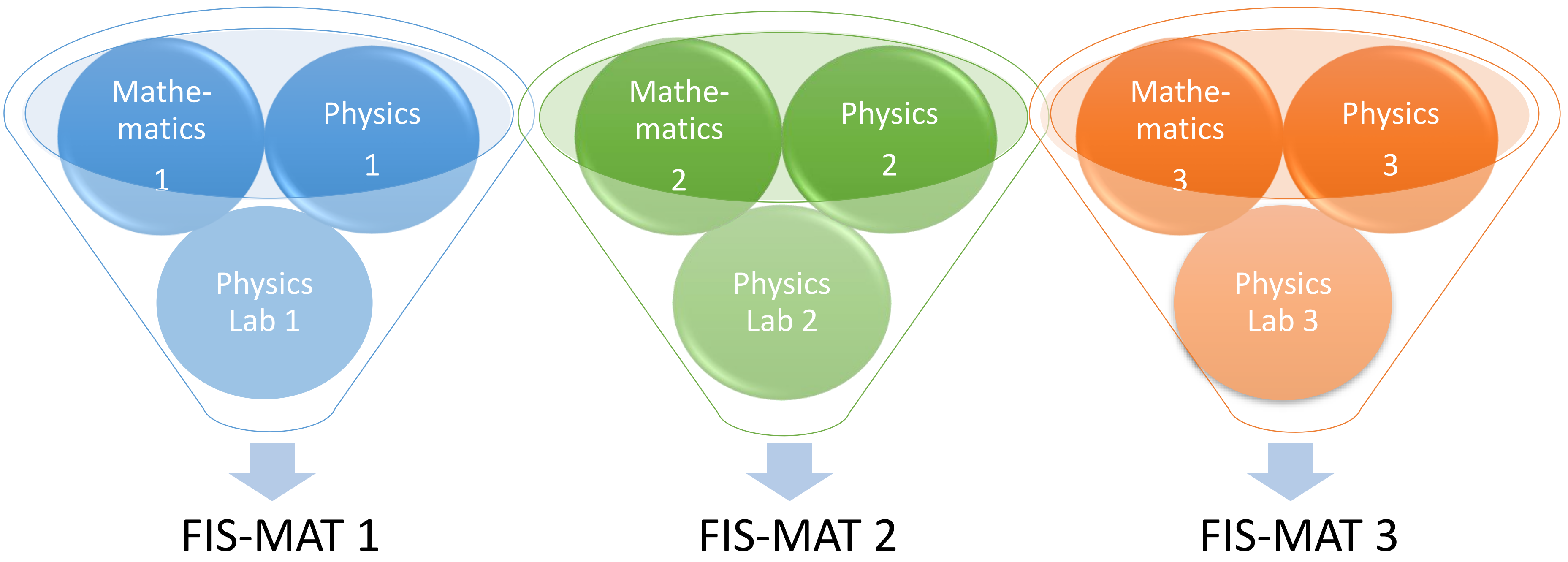
Tecnológico de Monterrey

Discussion and results

- Participant faculty members were pleased with the experience
- Students commented that the fact that boundaries between the physics and the math were lowered help them to better understand the application and need for the math content.
- Students added that the use of the technology gave them the opportunity to design, explore, investigate, experiment, analyze real data, interpret, discuss, and make connections.

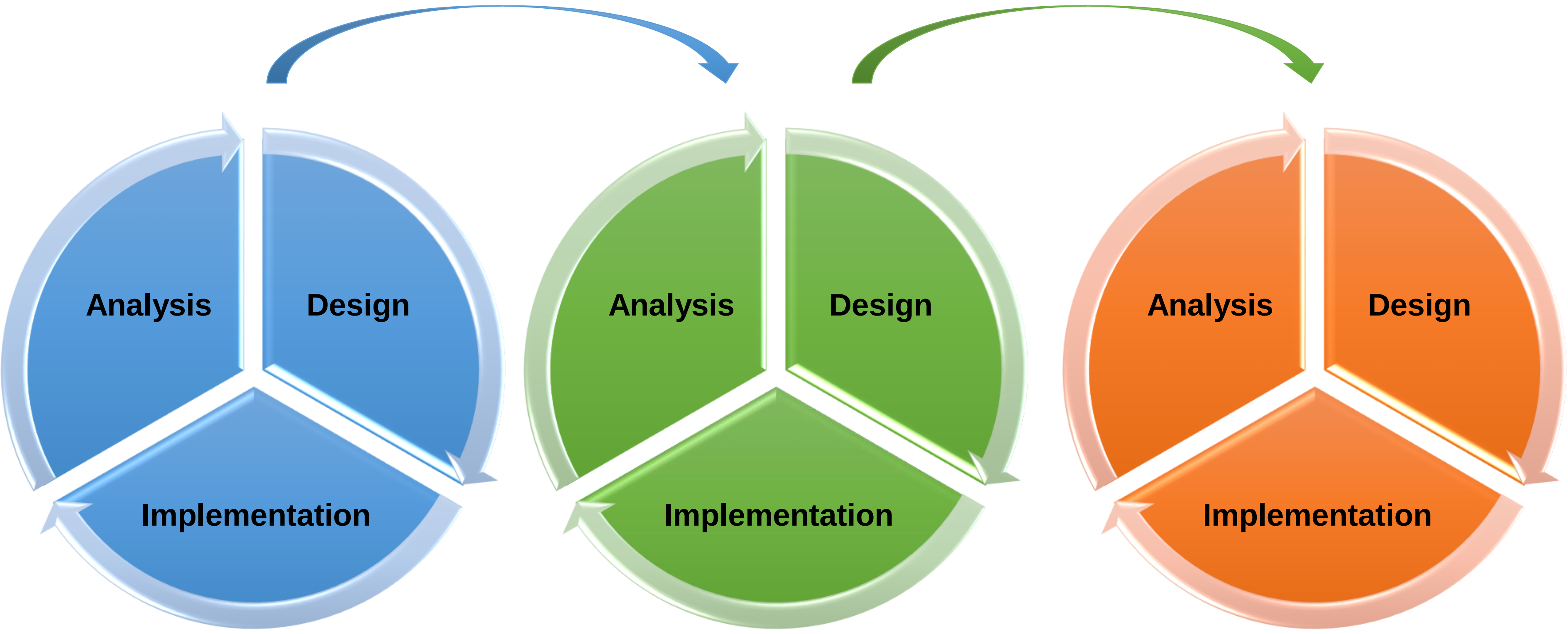
Curricular sequence

Three integrated courses of Physics and Mathematics (Fis-Mat 1, 2 and 3)



Next steps

Complete a sequence of three Fis-Mat courses corresponding to the first three courses of Physics and the three Calculus courses for engineering students.



Fis- Mat 1

Fis- Mat 2

Fis- Mat 3

Goals of the Fis-Mat Content

- Improve students' abilities to make connections between physics and mathematics.
- Deliver a flexible course that provides students with educational tools to help them over the conceptual difficulties.
- Foster and deeper understanding of the physical and mathematical concepts applied in engineering practices.

Goals of the Fis-Mat Pedagogy

- Introduce students to the engineering way of thinking to approach problems
- Create a learning environment
- Increase students' motivation
- Develop successful learners by helping students to become knowledgeable
- Align assessment (formative and summative)

Thank you

angeles.dominguez@itesm.mx

Tecnológico
de Monterrey