Cardano Parameters Modelling

EXECUTIVE SUMMARY

The paper ultimately explains why **the release of Reserves is less than 0.2%** even if the monetary expansion parameter is ρ=0.3%, and **the proportion between Rewards and Treasury is not 80% to 20%**, even if the treasury cut is τ=20%.

The analysis starts from two technical parameters with a huge tokenomic impact

- **Parameter k**, which is the desired number of pools determining the saturation level, i.e. the stake above which no additional rewards are paid.
- **Parameter $\boldsymbol{a_0}$**, which is the pledge influence, deciding the effect of the pledge (own stake provided by pool operators and not by delegators) on Rewards

The paper shows that $\boldsymbol{a_0}$ **reduces rewards also for smaller pools, not only for pools larger than saturation:** the benefit of the pledge is much lower for small and medium pools, compared to large ones near the saturation stake.

This depends also on k, since k determines the saturation stake:

$$\frac{Supply}{k} = Saturation \approx 70m\ ADA$$

Joint effect of a_0 and k and interactions of Size and Pledge:

The paper shows that only large pools have a clear reward increase for higher pledge, to the point that for pools smaller than $\frac{Saturation}{2} \approx 35M$ ADA, a larger pledge, at equal stake, can even reduce rewards.

In practice, the Reward formula gives incentives for pools to increase the stake till saturation, but the incentive to small pools is to increase the stake first by delegation.

- Lower k and higher a_0 discourage large owners from trying to game the system by splitting their stake into many pools, and reward large commitments of own stake.

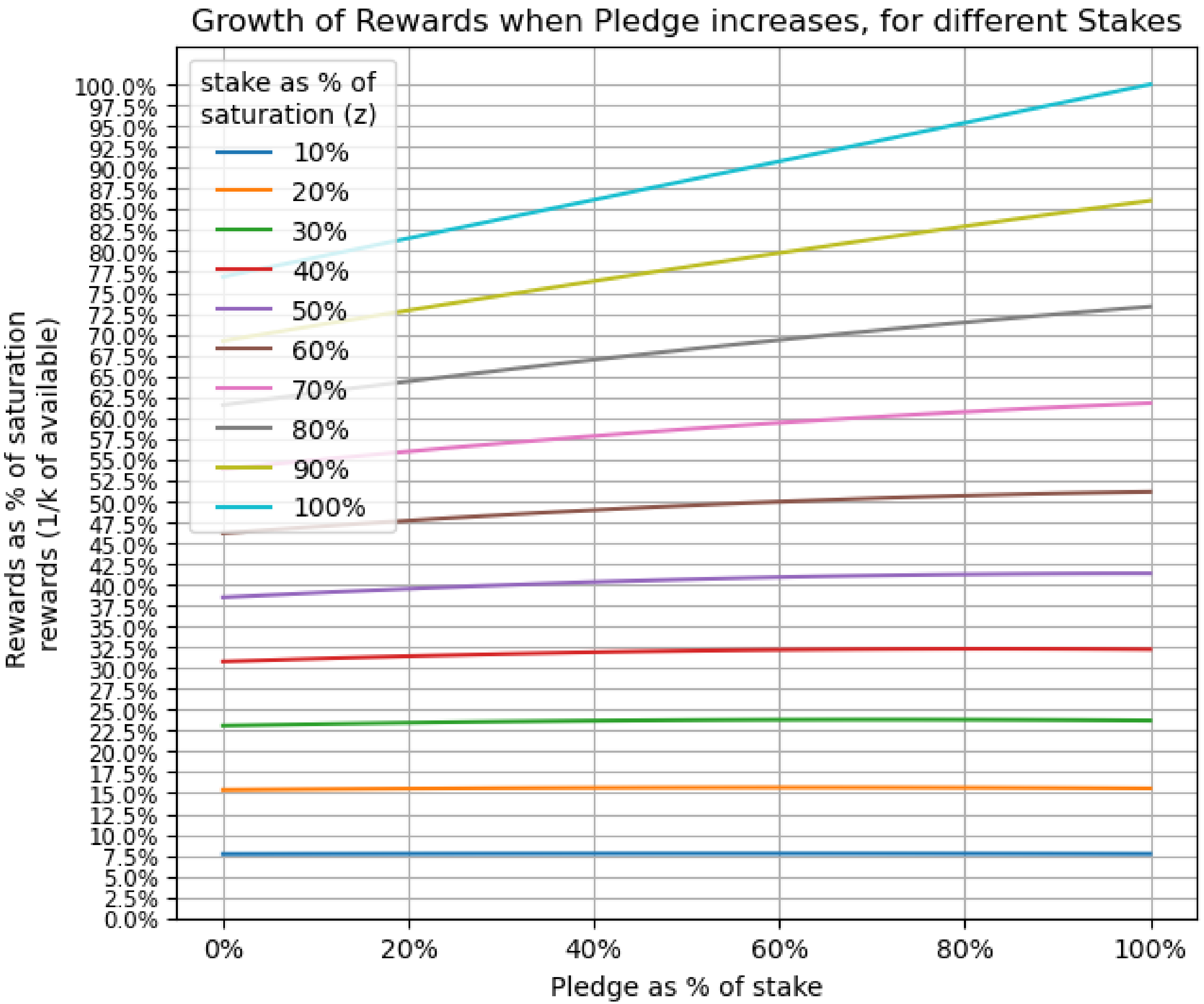
- But this can increase security and efficiency while reducing openness and decentralization, by reducing rewards for small pool operators and delegators

- Governance must keep balance through k and a_0 and be aware of the interactions. The paper provides several examples and analysis to this goal.

Joint Pledge and Size effects on Rewards:

Each line shows the rewards of pools with same stake size, but different percentages of pledge. The different lines represent different stake sizes.

We see for small-medium pools (stake at 10% of z means ~ 7M ADA) pledge has little positive effect, that even turns negative at high pledges. The latter effect vanishes for pools with a size of at least half of saturation.



Unpacking Reward Computations:

There are additional adjustments made to rewards, based on pool performance and active stake. While they have little effect on the total distribution of rewards, they help see that **reward adjustments can be split in two components**:

- one component, that we call **f**, is driven by incentive parameters a_0 and k
- the other one is just

$$\boldsymbol{p} = \frac{Active\ Stake}{Circulating\ Supply}$$

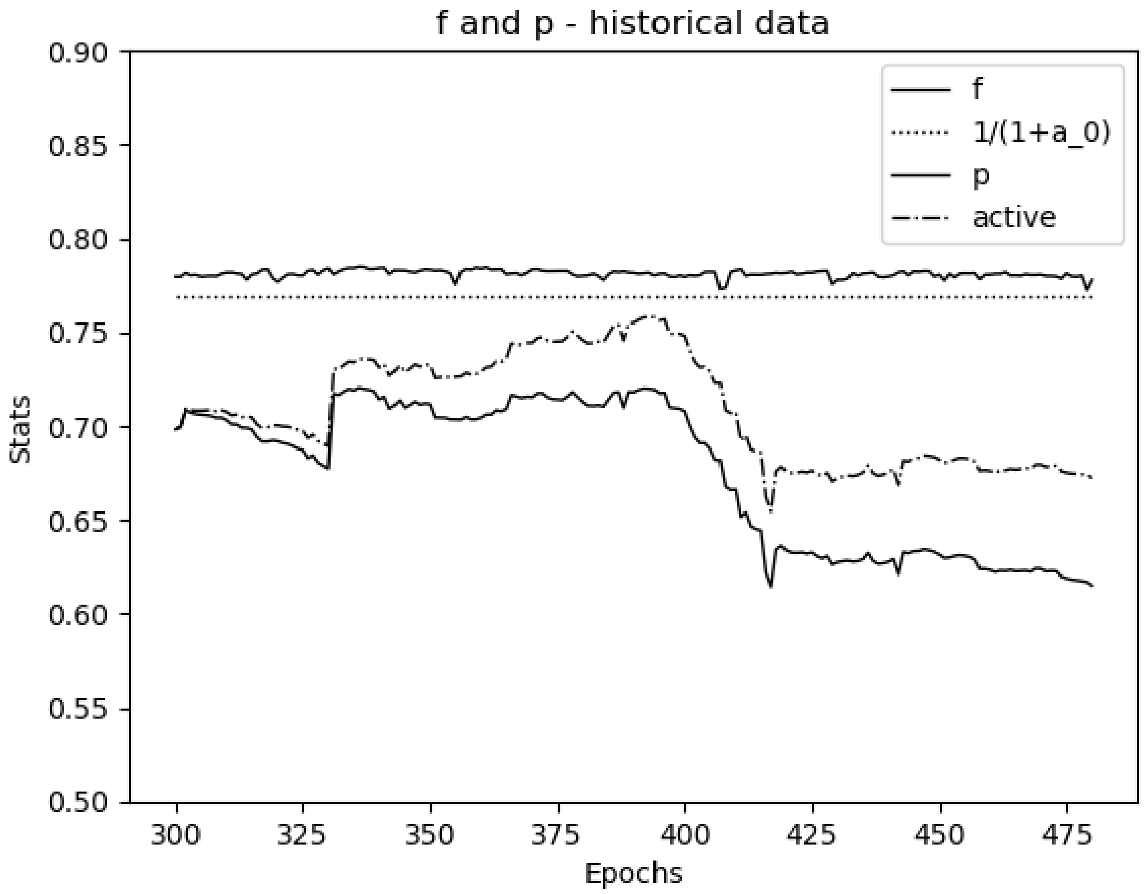
p component exists because the circulating supply is used as a measure of total stake for rewards, instead of active stake which is used in consensus.

Nex chart shows that **p** component reduces rewards more than **f**.

If the incentive formula computed stake as a percentage of the active stake, so that all stakes would sum to 1, as done in consensus, we would only reduce rewards by f.

The additional reduction is just a multiplication by p to consider that, in fact, stake is computed as a percentage of circulating supply, not active stake, in reward computations.

This makes reward behavior different from that of other chains, i.e. Bitcoin. It increases stability and predictability of the rewards of a single pool but reduces stability of the total amount of rewards.



This should be monitored by considering both the impact on the stability of the aggregate active stake, and the effect on the behavior of single validators.

Looking at stake and pledge granular data:

We have seen in the above analysis that **the f adjustment to rewards is in aggregate very stable and near to:**

$$\frac{\mathbf{1}}{\mathbf{1+a_0}} \approx \mathbf{77\%}$$

We analyze the actual composition of the staking pools of Cardano, to understand the interplay with the incentives seen above.

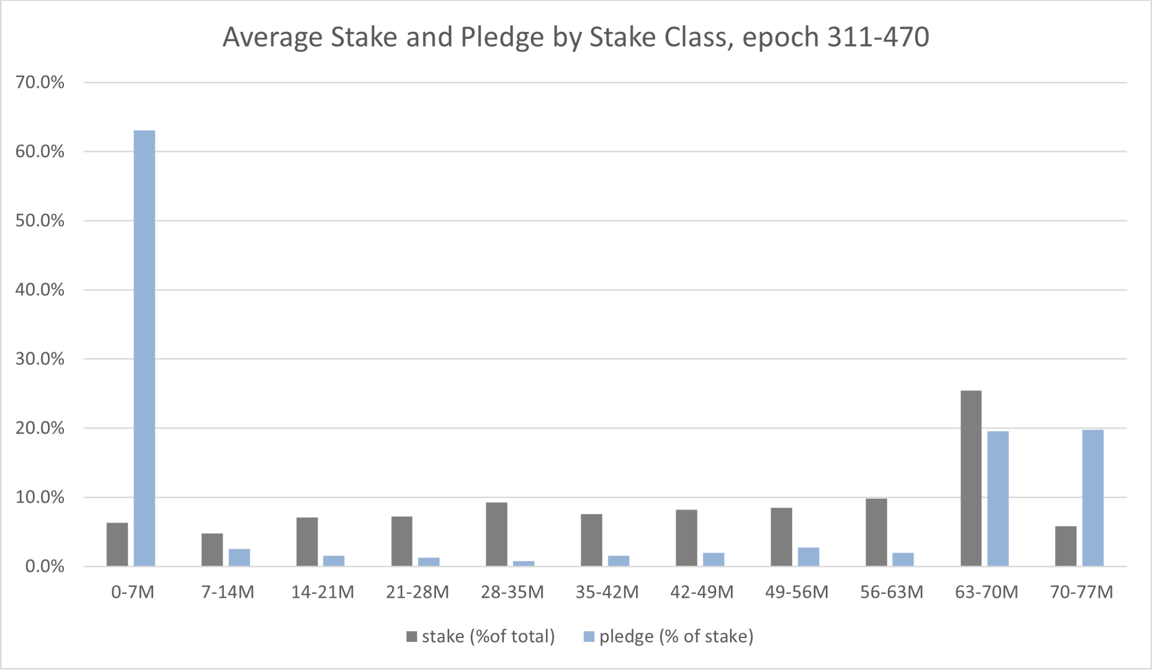
We find that the global amount of pledge is rather low, and that stake and pledge are unevenly distributed among pools of various sizes, with most of the stake – and of the portion in own stake - concentrated in the largest pools.

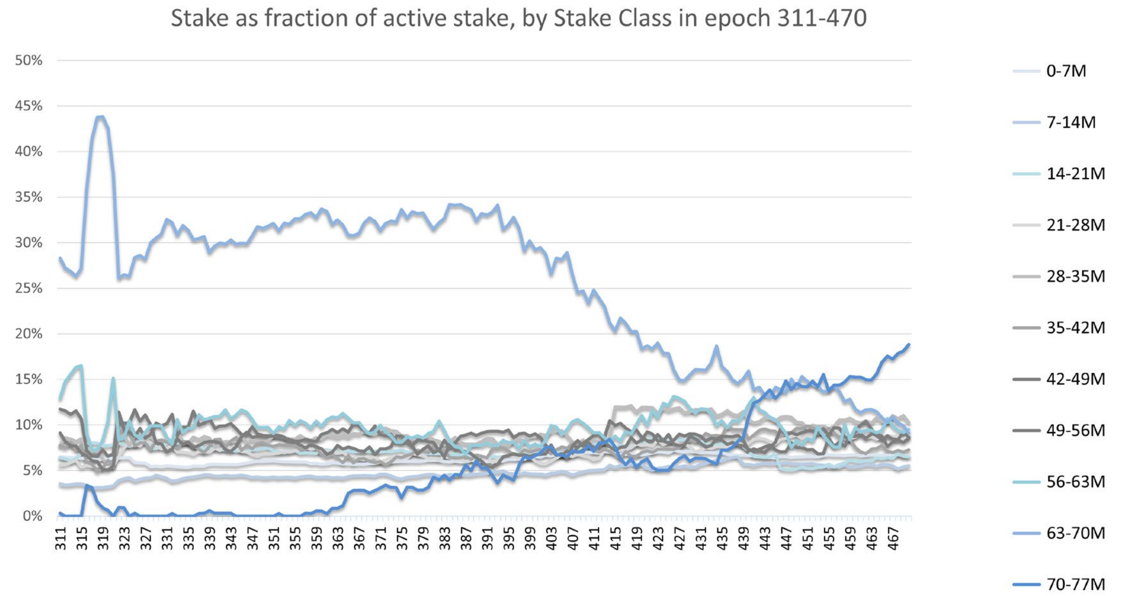
This explains f and is consistent with current parameterization of incentives.

We see in the top chart that % of pledge is higher for large pools than for medium ones. In the bottom chart we see that highest % of stake goes to largest pools.

This is consistent with pledge benefit being maximum for large pools, but not so high to bring very high pledge %.

Pledge as % is the highest for the smallest pools. This is because they start without delegates and need to attract them by showing commitment. But their impact remain small, since they contain less stake and have the smallest effect from pledge, as calculated in the paper in different ways.





The effect on release of Reserves and distribution to Rewards and Treasury

The incentives seen above influence the dynamics of the Reserves and their **distribution to Rewards and Treasury, making them very different from descriptions that overlook the interactions of parameters**.

The release of reserves every epoch does not trivially follow the monetary expansion parameter ρ=0.3%, nor the treasury expansion parameter τ=20% reveals the actual partition between Treasury increases and Rewards.

Due to the fraction which is retained from Rewards distribution, mostly as an incentive to grow stake and pledge or to rescale rewards by the active stake**, the actual release is smaller, and the relevance of Treasury over the total release, and in relation to Rewards, is higher.**

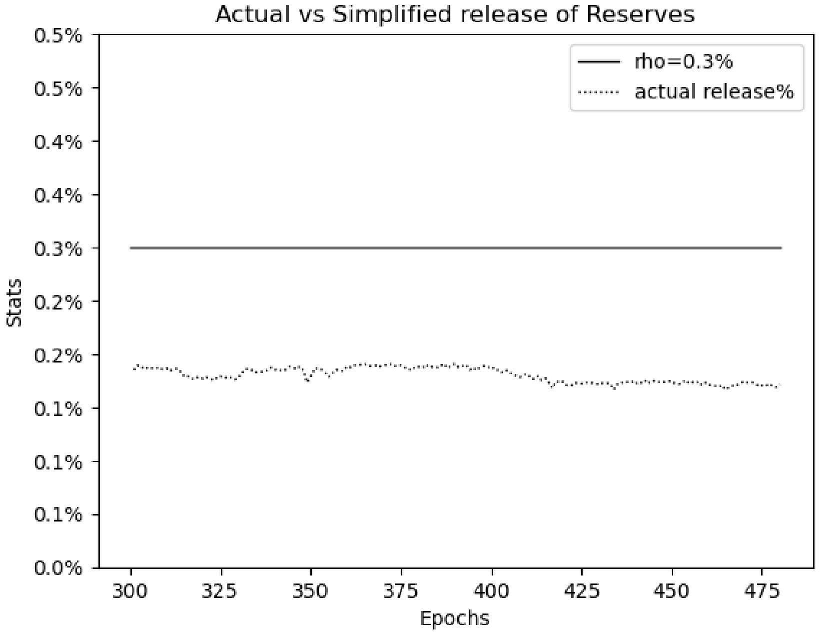
The actual release has been between **0.165% and 0.185%** rather than 0.3% (left chart)

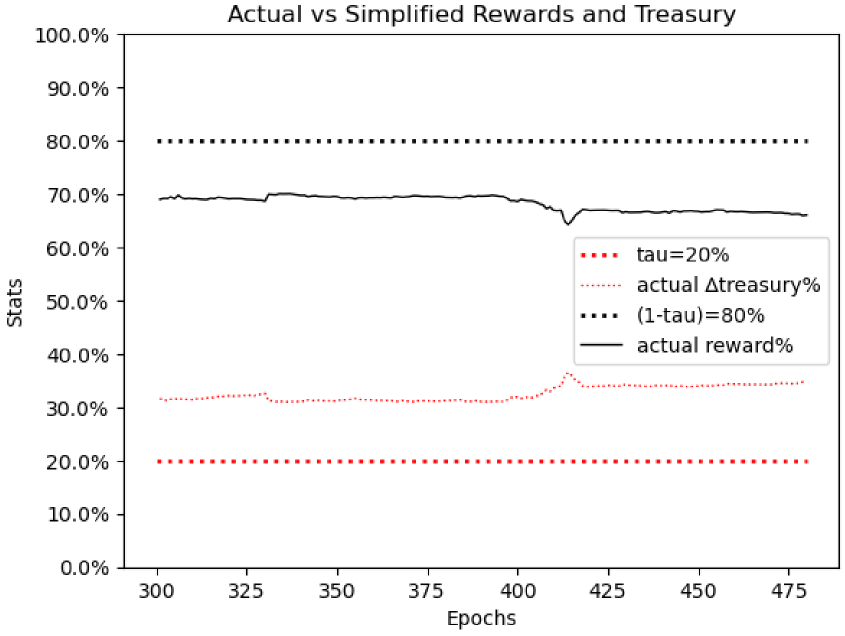
Rewards have been between a bit less than **65%** and little more than **70%** of the release, rather than 80%, with Treasury taking approximately **30%-35%** (center).

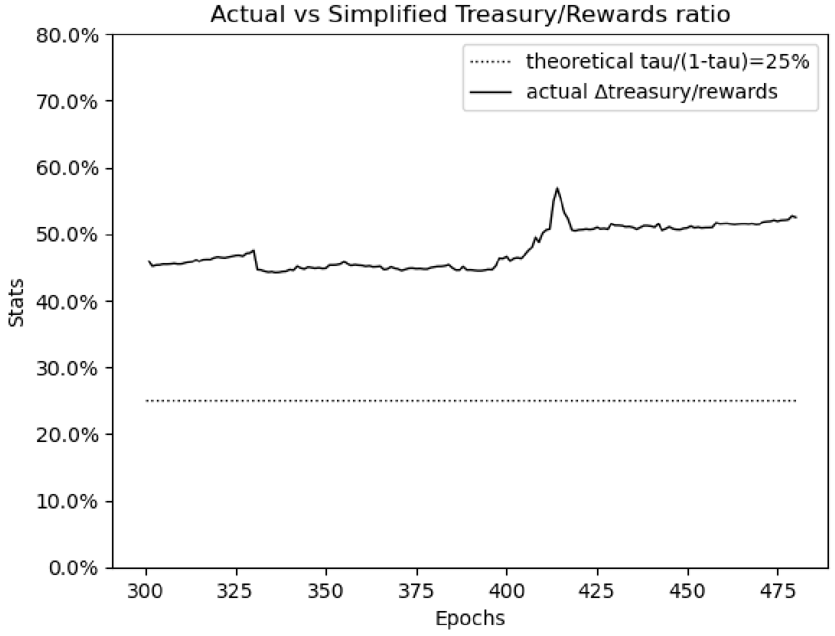
Thus the ratio between the growth of Treasury and the Rewards, given in principle by

$$\frac{\tau}{1-\tau} = 25\%$$

has often been more than **50%.**







Some conclusions:

The paper concludes by embedding the joint effects seen above in the equations governing the system dynamics. This shows the relevance of Rewards adjustments on Reserves projections (top chart), and also the limited impact of the higher-order terms in the incentive formula and of the current amount of transaction fees (bottom chart).

The paper analysis provides both foundations and insights for the stochastic modelling of the system future dynamics, required for simulations aware of the interactions of parameters and ecosystem variables, and able to embed the related probabilistic uncertainty.