Inspiring Teaching By Linking Mathematics to Industry

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1. Introduction

Mathematics at undergraduate level is often taught as a very abstract subject with no obvious connection to reality. Applied mathematics courses are often focused on mechanics, which tends to be regarded as a rather technical subject by students. As a result, undergraduate students often end up with a rather one sided view of mathematics, seeing it as a subject which is either completely useless, or as one in which applied maths is either poor maths, old fashioned maths, or not even real maths at all. As a consequence, when looking for careers students often think that the only possible jobs for mathematicians lie in teaching or in the financial sector. Sadly a similar view is held by many maths lecturers as well.

In fact nothing could be further from the truth! The reality is that mathematics lies at the heart of much of modern science and technology. Its applications are growing fast. Many of the real breakthroughs are now being made by applying what has until recently been considered pure maths to real world problems. And perhaps best of all for undergraduates, a degree in mathematics makes you extremely employable. Mathematicians are in great demand in industry, both for their subject knowledge and also for their problem solving skills.

From the perspective of teaching, using examples of industrial applications of mathematics has many advantages. Seeing how mathematics is used ‘in the real world’ can be extremely motivating for students who can sometimes be put off by an overly abstract approach to maths teaching. Industrial problems also have no respect for the borderlines between modules, and thus require students to think across boundaries, often using a variety of techniques from maths, stats and computing. Finally, industrial problems far from being easy applications of maths are usually very challenging and make students think out of the box to solve them. So, all in all, three very good reasons to put industrial examples into your teaching. There is also the advantage that they help in developing other skills such as team work, presentation skills, research skills, interdisciplinary working, modelling, working with unstructured problems and the skill of rapid computing.

2. Some examples of mathematical problems in industry

It is fair to say that mathematics is not only important in industry, but often is a driver in technological change. The recent Deloitte report on ‘The Importance of mathematics to the Economy’ made this very clear, indicating that mathematics made a contribution to the economy of upwards of £20 Billion to the UK.

Some examples of where mathematics has made a huge difference are

- *Google*, in which the Page Rank Algorithm relies on the calculation of the eigenvalues of very large matrices
- *Signal processing and digital technology*, which was a direct result of the development of the Fast Fourier Transform
- *Maxwell’s equations*, which led to radio, TV and the mobile phone
- *Error correcting codes* which make heavy use of finite fields
- *Cryptography* which uses number theory
- *Computer graphics* which makes heavy use of algebraic geometry, complex numbers and quaternions
- *Medical statistics*, with splendid links to Florence Nightingale
- *Medical imaging*, which uses linear algebra and tomography
- *Facebook*, Twitter, the Internet and social media which all use network theory.

There are of course many more, and the list is growing fast.

All of these are excellent motivational examples in a general teaching environment to show that the study of mathematics is important. (The Google example is especially nice as a way of introducing and motivating eigenvalues, and cryptography is used in Bath to motivate the teaching of number theory in the first year). An additional feature of all of these examples above is that they come with a nice human story. It is always worth telling students that mathematics is created by human beings.

It is interesting to note that the role and type of mathematics that is used in industry is also changing. If I were to be writing this article twenty years ago I would have said that the main industrial users of mathematics would have been in: telecommunication, iron & steel, oil, energy, defence, insurance, finance and weather forecasting. The sort of maths that would have been needed would primarily have involved calculus (especially differential equations), linear algebra and statistics. The situation is now very different with industrial users including: retail, food, zoos, sport, entertainment, media, forensic service, hospitals, air-sea-rescue, education, transport, risk, health, biomedical, environmental agencies, art, and anything involving people. These need a much wider range of mathematical tools including discrete maths, optimisation, number theory, graph theory, game theory, probability and topology. The introduction of fast and easy to use computer packages such as R, Matlab, Mathematica and Maple, rather than (as was predicted) killing of mathematicians has (in the same manner as the printing press for writers) liberated them, allowing them to do far more, engage with industry more easily and to make it far easier to bridge the gap between theory and practice.

3. How to link industrial examples into teaching

The challenge we therefore face in our teaching is how we can make industrial problems accessible to students, fun to do and achieve a worthwhile educational objective in the process. I believe (from my own direct experience) that there are three ways in which this can be done effectively.

1. Context Based Examples

Certain courses naturally allow the use of examples related to industry to be embedded directly within their content. Examples are courses in methods (eg. Calculus), mechanics, statistical inference, optimization and scientific computing. It is natural in such courses to give examples in the lectures which have their origin in industrial calculations, or to include such examples in problem sheets or in course work. In general such examples will be tightly defined and have a clear mathematical form. To give two examples:

(i) **Microwave cooking**

The equation

$$\frac{d^2T}{dx^2} + e^{-ax} = 0, \quad T(0) = 1, \quad T_x(1) = 0,$$

is an example of the sort of differential equation problem that might be encountered in the first year of an undergraduate course in calculus. It can be solved by the usual method of using a general solution and a particular integral. As such it is a fairly routine exercise which may not especially motivate a student. However, this example becomes much more relevant to students if you explain that if T is temperature then this is the equation for the temperature distribution inside a microwave cooker when you heat up a potato. (The exponential term here represents the loss of energy of the microwaves as they penetrate inside the potato). This is part of an industrial problem that I worked on for the food industry who wanted to find out how safe microwave cookers were (safe being measured in terms of how well they would kill bacteria). By solving the differential equation you can make estimates on how safe the cooker is. This is something every student who relies on a microwave cooker would like to know!

NOTE To make the problem more realistic you can replace the RHS by a time derivative of T. This is of course harder to solve, but also allows the students to see the progressive development of a model.

(ii) **Forensic Maths**

Most maths undergraduates do courses in mechanics (often compulsory in their first year). Maybe they don’t know that most students in forensic science courses also have to study mechanics! The reason for this is that a lot of mechanics is required to reconstruct either crime scenes (finding where bullets came from) or road traffic accidents (finding out what happened just prior to the accident and establishing whether a car was being driven dangerously). Given the current popularity of TV programmes such as CSI, students are often strongly motivated by problems in forensic science, and this gives a nice set of examples to use in mechanics. As an example, suppose that you measure the length of the skid marks left by a car in an accident, can you work out how fast the car was going prior to the accident? To solve this problem, students need to consider the various forces in action (eg. Braking force, road resistance etc.) and factors such as the amount of water on the road, slope of the road etc.

2. Course work based modules

There are certain courses which can be structured around industrial examples. These courses typically are not of the traditional lecture style, but usually involve group work and extended project based work often over a whole semester. These courses have the advantage of introducing the students to different learning styles, and also teach them the skills of team work, working under time pressure, extended writing and presentation skills. Such courses are most usually assessed by course work based on case studies. Students work together to solve extended problems based on industrial examples, write a ‘portfolio’ of their answers and often give a presentation. Examples of such courses are taught in Bristol, Bath, Greenwich Nottingham, Oxford and Reading at both undergraduate and graduate level. The case studies in the materials for today’s workshop could all (in principle) be usable in this context.

3. Focused days/weeks.

It is not always easy to devote an entire course to group work on industrial case studies, but it is quite possible to devote a day to this or perhaps a week of lectures can be used. This is a relatively painless way to have a realistic approach to solving ‘real’ industrial problems. One format (used in the physics department in Bath) is for a problem to be introduced on the first lecture of the week, and the students then have the rest of the week to work on it and to write a short report. Again the case studies we refer to in the course materials can be used in this context.

4. The study group experience

A powerful way of interacting with industry is through Industrial Study Groups. These started in the 1960s in Oxford and are now a huge world-wide activity. In a study group and industrialist presents problems on the first day of the week. Teams of academics and students then work on the problems for a week. On the final day they present their results, with a more considered report following later. Study groups as they currently exist are an excellent way of training PhD students in applied maths/stats/computation, and are also excellent training for academics. The study group problems can often lead to useful undergraduate case studies at a later date.

The website

http://miis.maths.ox.ac.uk/how/

gives lots of information on how the study groups work and the benefits that they bring. Study groups are often preceded by post graduate training campls in which students are given ‘sanitised’ industrial problems to work on, under the guidance of academics. See

http://miis.maths.ox.ac.uk/info/training.shtml

for more information.

5. Sources of good problems

One significant hurdle which can be raised against the use of industrial examples is that in their ‘raw’ state they are often too hard for students to tackle. The problems may be too vaguely defined for example, or in the general case they may be well beyond an undergraduate degree (for example solving a problem in turbulent combustion) requiring an immense amount of computer power. Usually to be able to present a problem to an undergraduate a certain amount of preliminary work must be done to simplify it to a point where undergraduate methods can be used, without losing the essence of the original industrial problem.

Fortunately there are a number of sources where the hard work has been done already. One is the book below which started life as a series of study group reports (see Section 4).

Practical Applied Mathematics: Modelling, analysis and approximation, by Sam Howison, published by CUP

Which, to quote from the reviewer

Drawing from a wide variety of topics to which mathematics is often applied "in the real world", the textbook tries to put "flesh on the bare bones of equations" in a way that will appeal to the student and the user. It is a nearly ideal introduction to the practical world of mathematical modelling for the neophyte, and the many applications are used to show that applied mathematics is much more than a series of "academic calculations".

Another very useful resource is the website

http://www.maths-in-industry.org/miis/

This gives a huge number of industrial case studies, ordered in terms of problem area, which have been worked through by a team of mathematicians and can mostly be used in the context of undergraduate teaching.

Another resource developed at the University of Bristol, funded by a grant from the Mathematical Sciences HE Curriculum Innovation Project, is the Industrial Problem Solving for Higher Education website

http://j.mp/ipshe

Again, this website gives a large number of industrial case studies that have been used in mathematical modelling units at the University of Bristol. The problems are classified in terms of difficulty as introductory, intermediate or advanced, which roughly translate as appropriate for 1st year, 2nd year or 3rd year students.

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