# Some principles which inform the way I teach

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# PART 1

4

5

2

8

6

9

7

1

3

$$3 + 6 = 9$$

4

5

2

8

6

9

7

1

3

$$3 + 6 = 9$$

$$\text{syv} + \text{to} = \text{åtte}$$

4

5

2

8

6

9

7

1

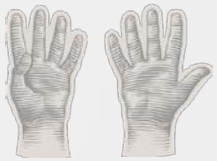
3

3 + 6 = 9

syv + to = åtte

4

tre

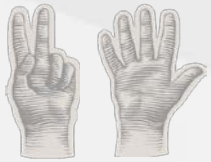


2

fire

5

seks



8

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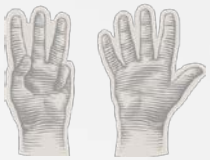
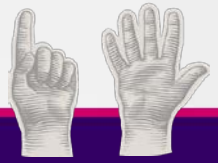


1

ni



3

syv


5
2
7
9
8
3
6
4
1

# Powers of the mind

- Counting is a complex activity
- We should not underestimate the *Powers of the Mind* (Gattegno, 1971) which all learners have
- Learning our first language is an amazing achievement...

  … and no-one ‘taught’ us this

# Powers of the mind

- Guiding:
  - Will
  - A sense of truth
- Working with ‘material’:
  - Creativity
  - Extraction
  - Abstraction
  - Association
  - Transformation
- Holding information:
  - Memory
  - Imagery
  - Functionalisation/Automaticity

Hewitt, D. (2015). The economic use of time and effort in the teaching and learning of mathematics. In S. Oesterle and D. Allan (Eds), *Proceedings of the 2014 Annual Meeting of the Canadian Mathematics Education Study Group*, (pp. 3-23). Edmonton, Canada: CMESG/GCEDM.

# Principles 1

- Call upon the range of students' *powers of the mind* and not just memory.
  - Tell them only those things which need to be told
  - The rest can become known through offering well designed tasks and carefully considered questioning

# TWO PROBLEMS INVOLVING 1, 2, 3, 4, 5, 6, 7, 8, 9

# Problem 1

- Use the digits 1 to 9 once only to make this true:

$$\_\ \_ \times \_\ \_\ \_ = \_\ \_\ \_\ \_$$

$$24 \times 315 = 7560$$

- 0 not allowed
- Repeat of 5
- No 8 or 9

How many ways can you find?

# Problem 2

Arrange the digits 1 to 9 to make a single 9-digit number.
For example:

382461795

# Problem 2

3

Multiple of 1?

# Problem 2

382461795

Multiple of 2?

# Problem 2

382461795

Multiple of 3?

# Problem 2

3824

Multiple of 4?

# Problem 2

38246

Multiple of 5?

# Problem 2

382461795

Multiple of 6?

# Problem 2

$$\underline{3824617}95$$

Multiple of 7?

# Problem 2

$$\underline{3824617}95$$

Multiple of 8?

# Problem 2

382461795

Multiple of 9?

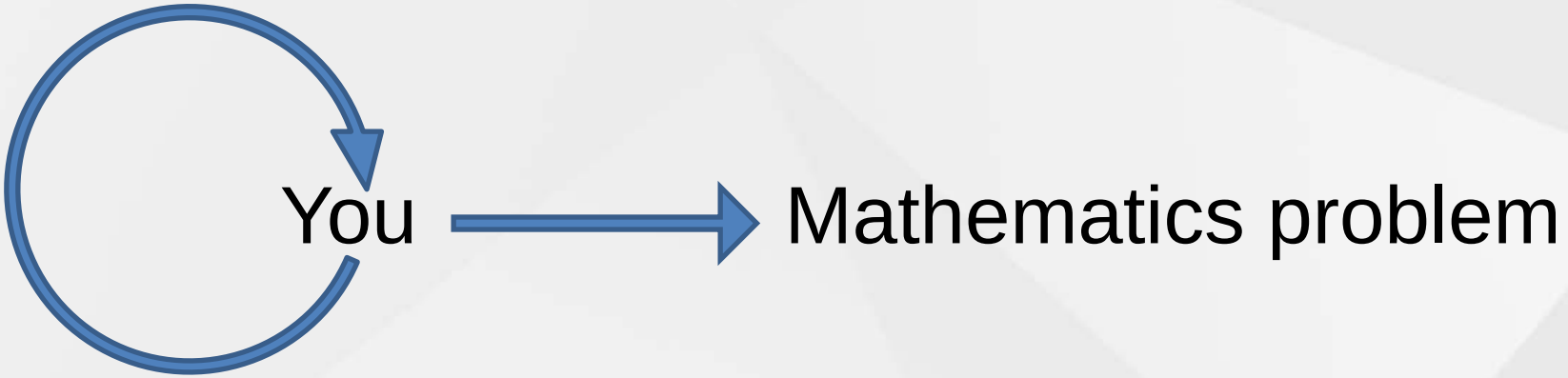
# 1, 2, 3, 4, 5, 6, 7, 8, 9

- Use the digits 1 to 9 once only to make this true:

  _ _ × _ _ _ = _ _ _ _

- Use the digits 1 to 9 to make a single number where:
  - First digit number is divisible by 1;
  - First two digit number is divisible by 2;
  - First three digit number is divisible by 3;
  - Etc….

# Way of working



# 1, 2, 3, 4, 5, 6, 7, 8, 9

- Use the digits 1 to 9 once only to make this true:

  _ _ × _ _ _ = _ _ _ _

- Use the digits 1 to 9 to make a single number where:
  - First digit number is divisible by 1;
  - First two digit number is divisible by 2;
  - First three digit number is divisible by 3;
  - Etc….

# What is important?

- Not the answer!
- The mathematics you:
  - become aware of whilst working on the problem;
  - are practising whilst working on the problem.

Problem 1:

- Awareness of how many digits an answer might have
- Knowing the final digit to a calculation

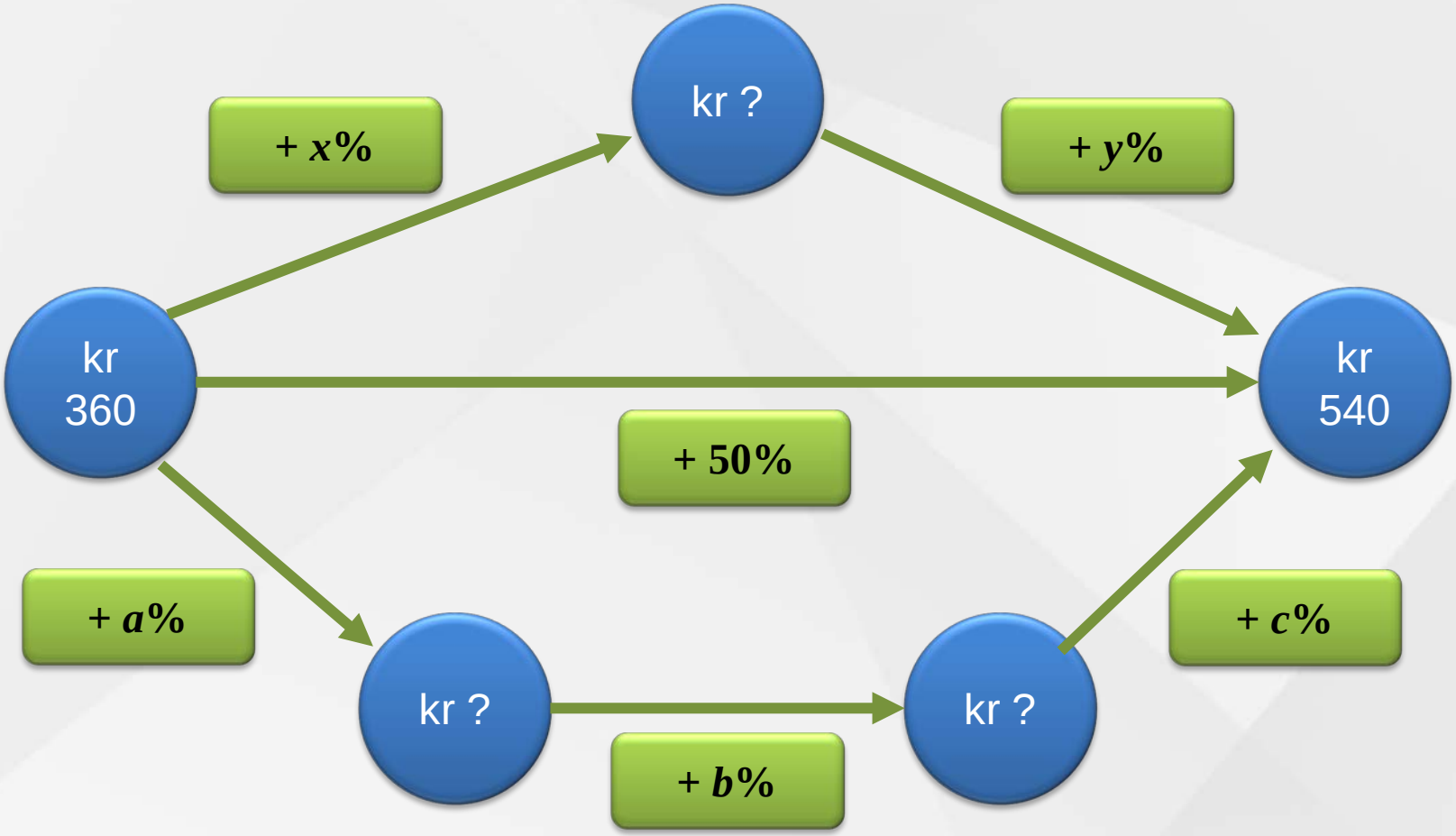
Problem 2:

- Properties of multiples of 2, 3, 4, 5, etc.

# Principles 2

- For an educator, answers are not as important as what awarenesses are gained whilst working on a problem
- Students should be working harder on the mathematics than their teacher

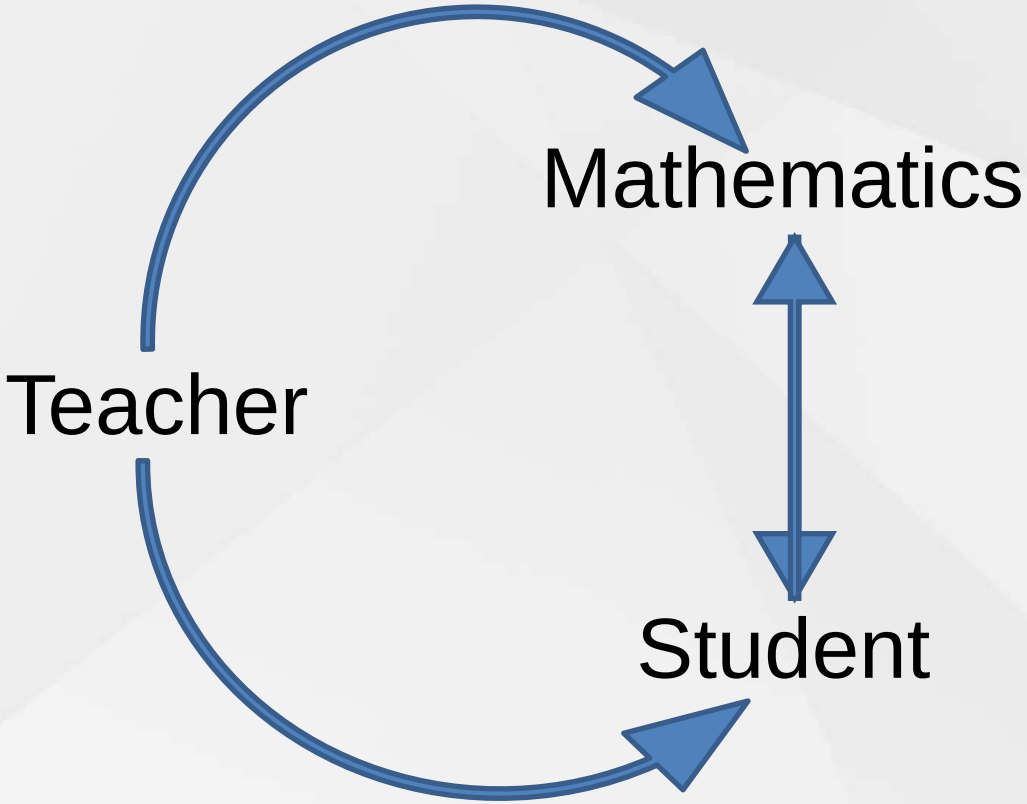
# Practising



# Principles 3

- Practise through progress
  - offer a challenge which requires lots of practice *and* opportunities to learn more at the same time.

# Principles



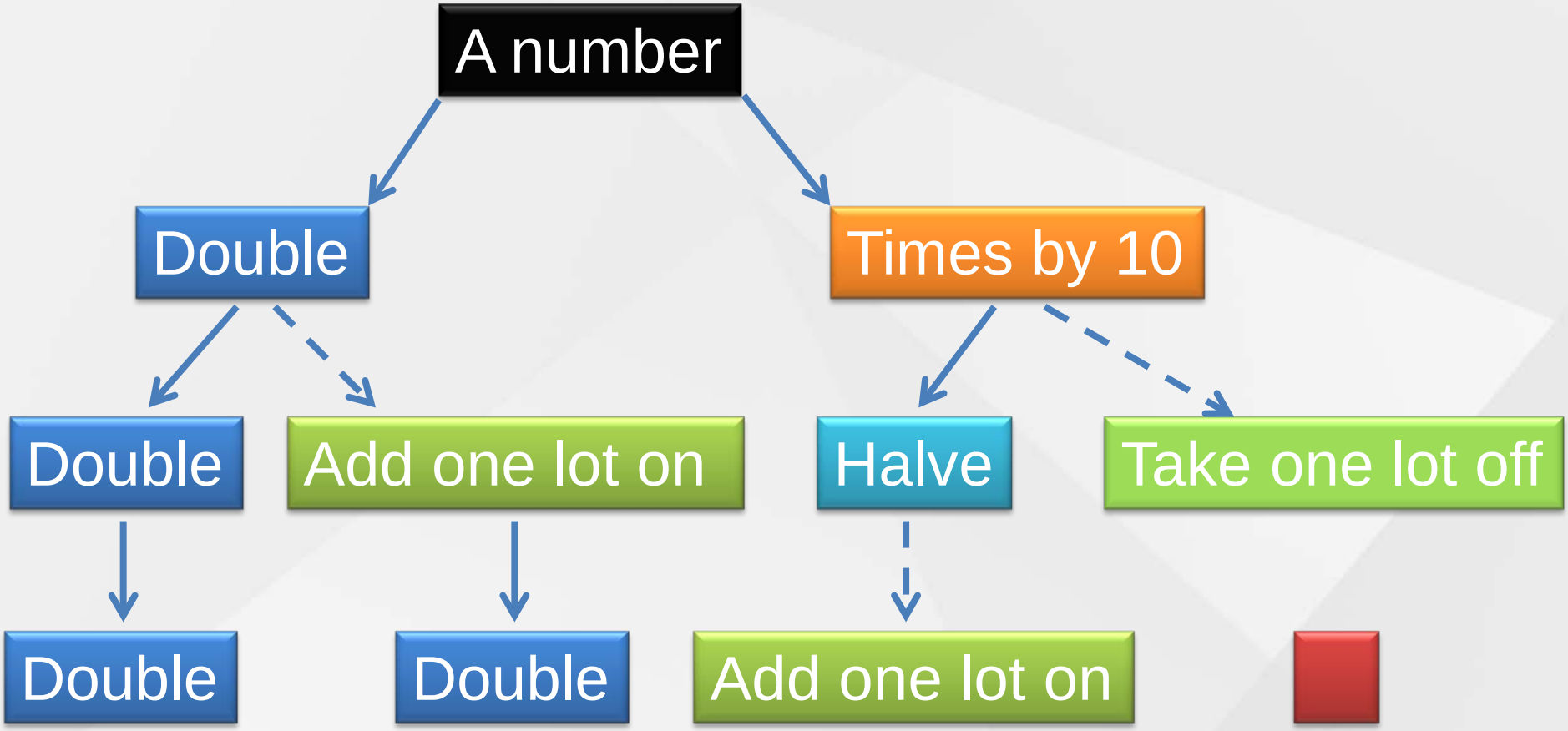
# Principles 4

- Less teacher ‘explaining’, more students ‘noticing’
- Teacher’s attention is with the student’s mathematical thinking, whilst the student’s attention is with the mathematics

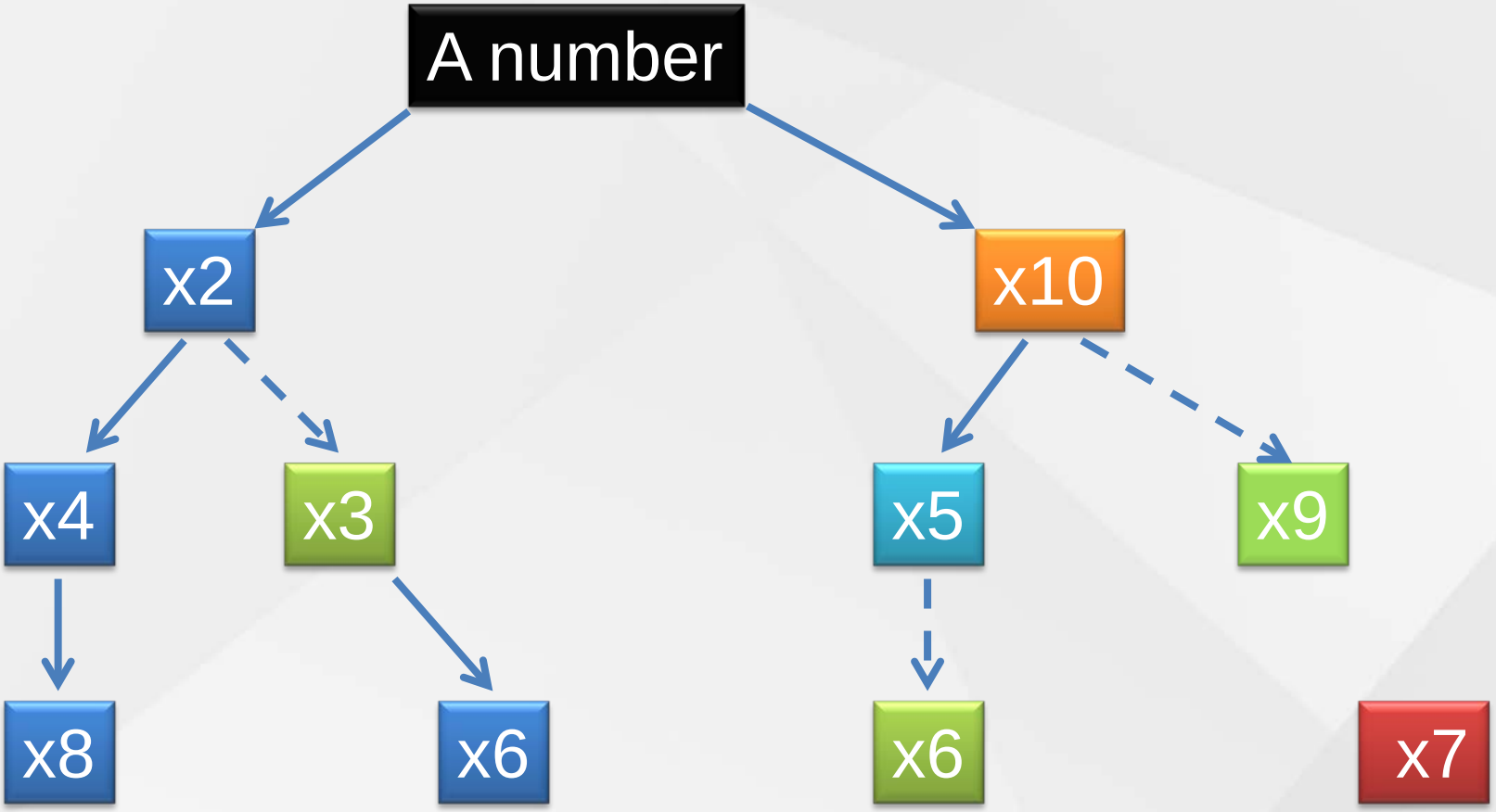
# END OF PART 1

# PART 2

# Operations


A number
Double
Times by 10
Double
Add one lot on
Halve
Take one lot off
Double
Double
Add one lot on

# Operations



# Times tables

What is required:

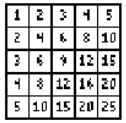
- Doubling and halving
- Adding or subtracting ‘one lot’
- Multiplying by 10
- 7 x 7 = 49

# Principles 5

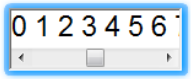
- Find an approach to a topic which requires the least ‘prior knowledge’ for a student (Direct access)

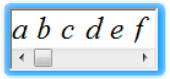
# Grid Algebra

# Grid Algebra

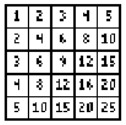



| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 |
| 2 | 2 | 4 | 6 | 8 | 10 |
| 3 | 3 | 6 | 9 | 12 | 15 |
| 4 | 4 | 8 | 12 | 16 | 20 |
| 5 | 5 | 10 | 15 | 20 | 25 |
| 6 | 6 | 12 | 18 | 24 | 30 |

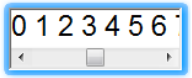


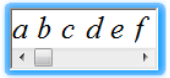



# Grid Algebra

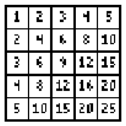



| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 2 | $5-2$ | 4 | 5 |
| 2 | $\frac{4(5-2)-8}{2}$ | 4 | 6 | $\frac{4(5-2)-8}{2}+6$ | 10 |
| 3 | 3 | 6 | 9 | 12 | 15 |
| 4 | $4(5-2)-8$ | 8 | $4(5-2)$ | 16 | 20 |
| 5 | 5 | 10 | 15 | 20 | 25 |
| 6 | 6 | 12 | 18 | 24 | 30 |

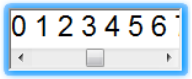


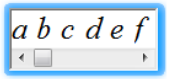



# Grid Algebra

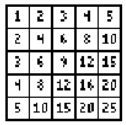



| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 |
| 2 | 4(5−2)−8 | 4 | 5 | 4(5−2)−8 / 2 +6 | 10 |
| 3 | 3 | 6 | 9 | 12 | 15 |
| 4 | 4(5−2)−8 | 8 | 4(3−2) | 16 | 20 |
| 5 | 5 | 10 | 15 | 20 | 25 |
| 6 | 6 | 12 | 18 | 24 | 30 |

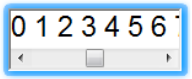


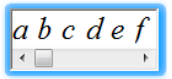



# Grid Algebra

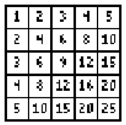



| | | | | | |
|---|---|---|---|---|---|
| 1 | | | $5-2$ | | $5$ |
| 2 | $\frac{4(5-2)-8}{2}$ | | | $\frac{4(5-2)-8}{2}+6$ | |
| 3 | | | | | |
| 4 | $4(5-2)-8$ | | $4(5-2)$ | | |
| 5 | | | | | |
| 6 | | | | | |

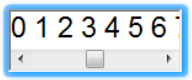


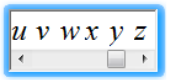



# Grid Algebra: what was the journey?

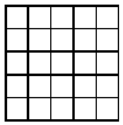



| | | | | | |
|---|---|---|---|---|---|
| 1 | | | | | 5 |
| 2 | | | | $\frac{4(5-2)-8}{2}+6$ | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |

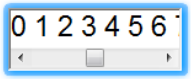


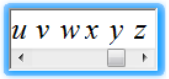



# Grid Algebra: the ‘same’ problem?




| | | | | | |
|---|---|---|---|---|---|
| 1 | | | | | $x$ |
| 2 | | | | $\frac{4(x-2)-8}{2}+6$ | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |



# Grid Algebra: what number should be entered into the blue cell?

| | | | | | |
|---|---|---|---|---|---|
| 1 | | | $x-2$ | | $x$ |
| 2 | $\frac{4(x-2)-8}{2}$ | | | $\frac{4(x-2)-8}{2}+6$ | |
| 3 | | | | | |
| 4 | $4(x-2)-8$ | | $4(x-2)$ | | |
| 5 | | | | | |
| 6 | | | | | |

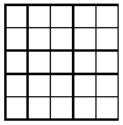
Tools
Menu
1..2 a..z
3×2

Magnifier
$x = 25$

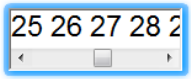
25 26 27 28 2

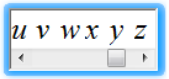
$u$ $v$ $w$ $x$ $y$ $z$

# Grid Algebra: where was $x$?
# Inverse journey

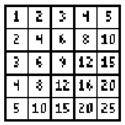



| | | | | | |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | $\frac{4(x-2)-8}{2}+6$ | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |



# Grid Algebra: taking 38 on inverse journey




| | | | | | |
|---|---|---|---|---|---|
| 1 | | | $x-2$ | | $x$ |
| 2 | $\frac{4(x-2)-8}{2}$ | | | 38 | |
| 3 | | | | | |
| 4 | $4(x-2)-8$ | | $4(x-2)$ | | |
| 5 | | | | | |
| 6 | | | | | |

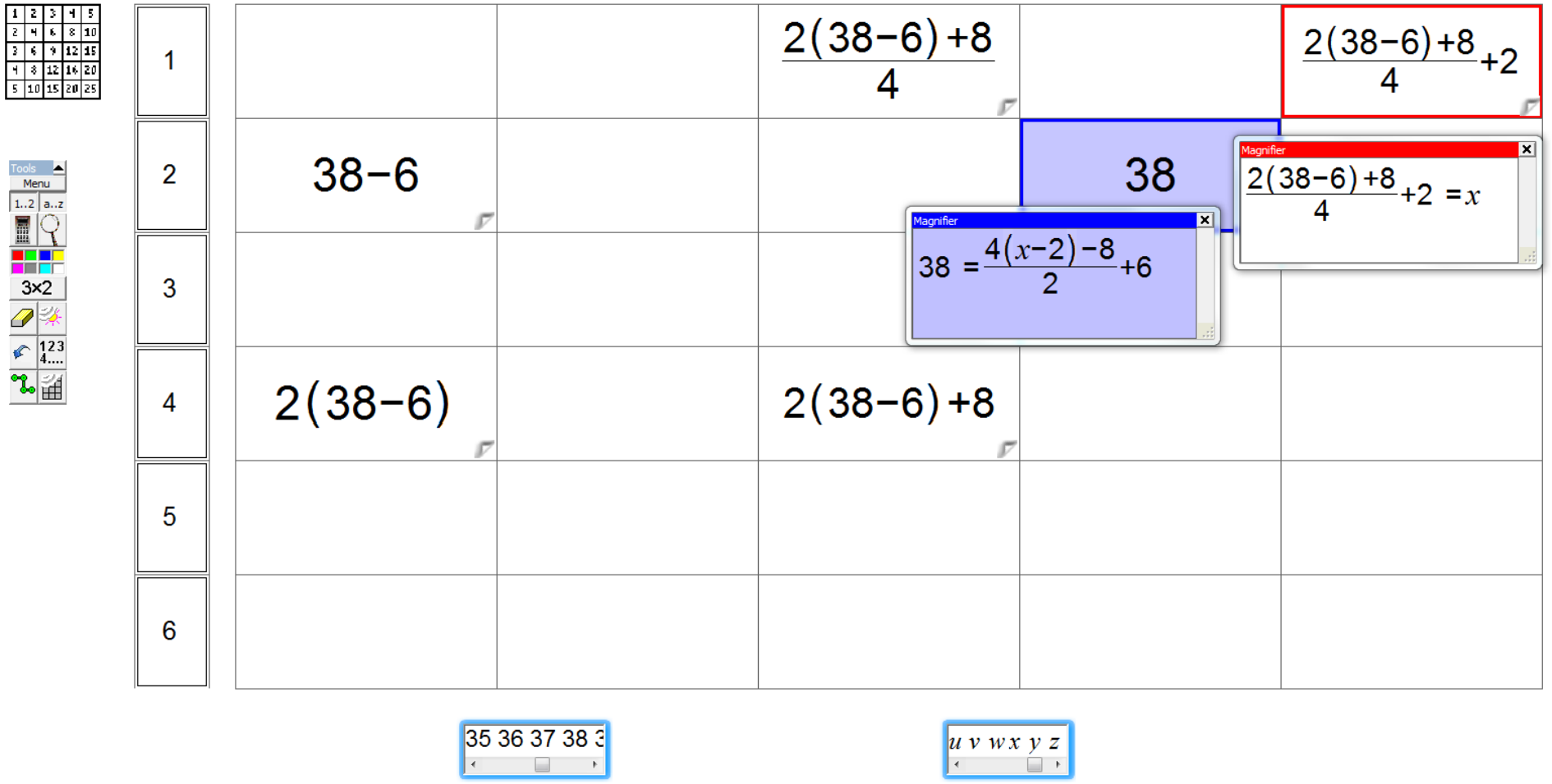
Magnifier

$$38 = \frac{4(x-2)-8}{2} + 6$$

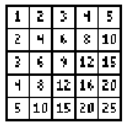
35 36 37 38 3

$u\ v\ w\ x\ y\ z$

# Grid Algebra: inverse journey produces how to solve the equation



# Grid Algebra: calculation carried out and correct answer entered into cell




| | | | | | |
|---|---|---|---|---|---|
| 1 | | | $\frac{2(38-6)+8}{4}$ | | $x$ |
| 2 | $38-6$ | | | $38$ | |
| 3 | | | | | |
| 4 | $2(38-6)$ | | $2(38-6)+8$ | | |
| 5 | | | | | |
| 6 | | | | | |

Magnifier
$38 = \frac{4(x-2)-8}{2}+6$

Magnifier
$x = 20$
$= \frac{2(38-6)+8}{4}+2$

20 21 22 23 2

$u\ v\ w\ x\ y\ z$

# Grid Algebra: students aged 9-10

For each question write down the inverse journey, starting at the final number in given equation, to end up back at the letter. Then find the value of the letter in each case.

Question 1:

$2(t+3)=46$

Question 2:

$\frac{k-8}{4}=17$

Question 3:

$3v+12=132$

Question 4:

$\frac{2t-8}{4}=41$

Question 5:

$\frac{f-8}{2}+6=57$

Question 6:

$4\left(\frac{z}{2}+3\right)=68$

Question 7:

$\frac{2(w+6)-18}{3}=86$

Question 8:

$3\left(\frac{s}{5}-1\right)+9=102$

Question 9:

$2\left(\frac{b-18}{3}+4\right)=128$

Question 10:

$\frac{3d-18}{6}+1=37$

Question 11:

$5\left(\frac{g-6}{3}+1\right)+10=85$

Question 12:

$\frac{2\left(\frac{x}{3}-4\right)+12}{4}=39$

Question 13:

$2\left(\frac{3h+18}{2}-12\right)=138$

Question 14:

$\frac{4(y-3)+16}{2}-6=98$

Question 15:

$2\left(3\left(\frac{e}{5}-1\right)+9\right)=144$

Question 16:

$3(2(j-3)+8)-12=174$

Question 17:

$4\left(\frac{5p-15}{5}+1\right)+12=132$

Question 18:

$4\left(\frac{\frac{x+18}{2}-6}{3}+3\right)=128$

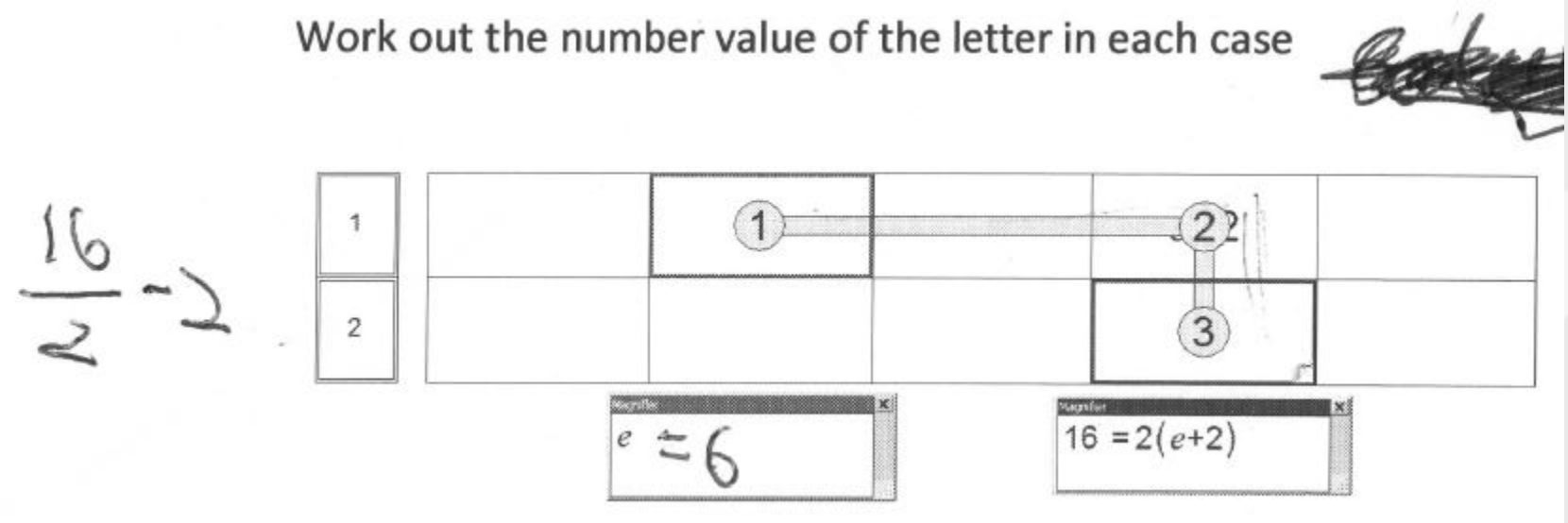
Question 19:

$3\left(2\left(\frac{k}{3}+4\right)-6\right)+12=114$

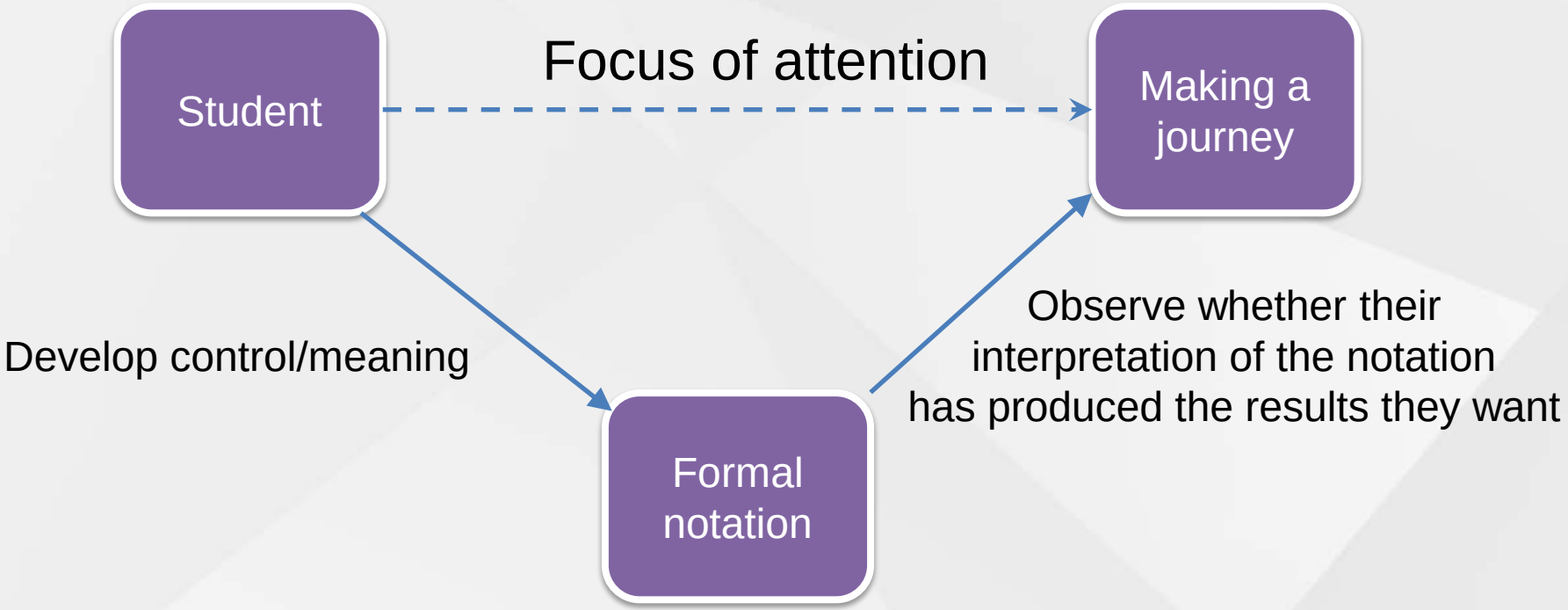
Question 20:

$\frac{3\left(\frac{4r-8}{2}+6\right)-24}{6}+2=43$

# Grid Algebra: students aged 9-10



# Subordination




Hewitt, D. (1996). Mathematical fluency: the nature of practice and the role of subordination. *For the Learning of Mathematics 16*(2), pp. 28-35.

# Principles 6

- Subordination helps students become fluent quickly

# Grid Algebra

Grid Algebra calls upon students' experiences of making journeys

- Order:
  - they go down one road before turning into the next road
- Inverse:
  - coming back requires going in the opposite direction and undoing the order "last part first"

# Principles 7

- Students have experiences from their everyday lives which are fundamentally mathematical in nature.
  - Try to call upon experiences from their lives which relate to the mathematics

# One statement?

$200 + 300 = 500$

$2cm + 3cm = 5cm$

$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

$5 = 2 + 3$

$3 + 2 = 5$

$2 + 3 = 5$

$20 + 30 = 50$

$2(7x - 4) + 3(7x - 4) = 5(7x - 4)$

$5 - 2 = 3$

$2x + 3x = 5x$

$5 - 3 = 2$

$0.2 + 0.3 = 0.5$

$2000 + 3000 = 5000$

# If I know this, what else do I know?

In pairs or threes:

- Choose *one* of the statements below
- Take turns to try to say as many different statements as you can which come from this one statement
- The connection with the statement must be clear!
- Do not carry out any of the operations in the statement

$$25 \times 4 = 100$$

$$\sqrt{64} = 10 - 2$$

$$2(4 + 1) = 10$$

# Principles 8

- Get a lot from a little

# Principles - overview

- Call upon the range of students' *Powers of the Mind* and not just memory.
  - Tell them only those things which need to be told.
  - The rest can become known through offering well designed tasks and carefully considered questioning
- For an educator, answers are not as important as what awarenesses are gained whilst working on a problem
- Students should be working harder on the mathematics than their teacher
- Practise through progress
  - offer a challenge which requires lots of practice *and* opportunities to learn more at the same time.

# Principles - overview

- Less teacher ‘explaining’, more students ‘noticing’
- Teacher’s attention is with the student’s mathematical thinking, whilst the student’s attention is with the mathematics
- Find an approach to a topic which requires the least ‘prior knowledge’ for a student (Direct access)
- Subordination helps students become fluent quickly
- Students have experiences from their everyday lives which are fundamentally mathematical in nature.
  - Try to call upon experiences from their lives which relate to the mathematics
- Get a lot from a little

# Principles - overview

- All these principles relate to the notion of *economic use of personal time and effort in the teaching and learning of mathematics*

# References and software


**Reference relating to the notion of *economy***

- Hewitt, D. (1994). *The principle of economy in the teaching and learning of mathematics.* Unpublished PhD dissertation. The Open University, Milton Keynes.

**References relating to *Powers of the Mind*:**

- Hewitt, D. (2015). The economic use of time and effort in the teaching and learning of mathematics. In S. Oesterle and D. Allan (Eds), *Proceedings of the 2014 Annual Meeting of the Canadian Mathematics Education Study Group, (pp. 3-23). Edmonton, Canada: CMESG/GCEDM.*
- Gattegno, C. (1971). *What we owe children. The subordination of teaching to learning.* London: Routledge and Kegan Paul Ltd.

**Reference relating to subordination:**

- Hewitt, D. (1996). Mathematical fluency: the nature of practice and the role of subordination. *For the Learning of Mathematics 16*(2), pp. 28-35.


***Grid Algebra***

- *Grid Algebra* is available from the Association of Teachers of Mathematics
- https://www.atm.org.uk/Shop/Primary-Education/Software-Media/Grid-Algebra---Single-User-Licence/sof071

# References relating to Grid Algebra


- Hewitt, D. (2016). Designing educational software: the case of Grid Algebra. *Digital Experiences in Mathematics Education 2(2)*, pp. 167-198. DOI: 10.1007/s40751-016-0018-4.
- Hewitt, D. (2014). A symbolic dance: the interplay between movement, notation, and mathematics on a journey toward solving equations. *Mathematical Thinking and Learning 16(1),* pp. 1-31. DOI: 10.1080/10986065.2014.857803.
- Hewitt, D. (2013). Learning algebraic notation and order of operations using Grid Algebra software. *Mathematics Teaching 232*, pp. 21-24.
- Hewitt, D. (2013). Introduction of letters and solving linear equations using Grid Algebra. *Mathematics Teaching 233*, pp. 6-10.
- Hewitt, D. (2012). Young students learning formal algebraic notation and solving linear equations: are commonly experienced difficulties avoidable? *Educational Studies in Mathematics 81(2)*, pp. 139-159. DOI: 10.1007/s10649-012-9394-x.

# Resources relating to Grid Algebra

- ‘Videos’  relating to how Grid Algebra can be used in the classroom can be found on YouTube.

- Search in YouTube under ‘Grid Algebra’ and ‘Dave Hewitt’

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