Hound and Hare

A hound starts in pursuit of a hare at a distance of 30 of his own leaps from her. He takes 5 leaps while she takes 6 but covers as much ground in 2 as she in 3. In how many leaps of each will the hare be caught?

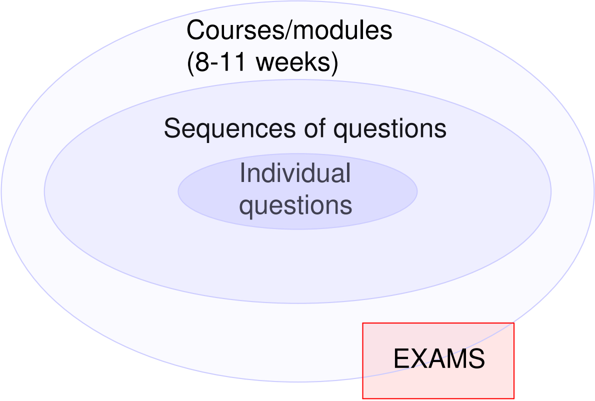
Mathematical tasks: past, present and future

Chris Sangwin

School of Mathematics
University of Edinburgh

October 2017

Outline


Courses/modules
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Tuckey, C. O., *Examples in Algebra*, Bell & Sons, London, (1904) Ex 64, (44).

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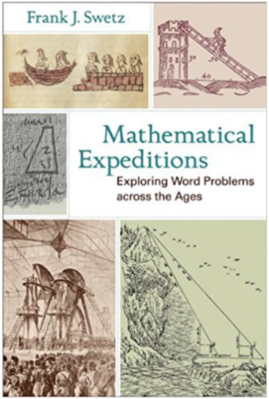

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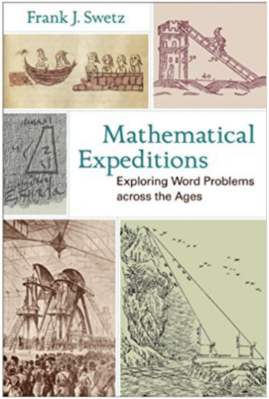

- Alcuin of York *Problems to Sharpen the Youth.* (790CE)
- *Jiuzhang suanshu*, Chapter 6, problem 14.
  (A collection of 247 problems from 100CE)

The problem as artefact

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Problems as “mathematical verse”.

Well-versed

A phrase meaning

> *highly experienced, practiced, or skilled; very knowledgeable; learned.*

Patterns of thought

Alice looks at Bob and Bob looks at Clare.
Alice is married but Clare is not.
Prove that a married person looks at an unmarried person.

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$\sqrt{2}^{\sqrt{2}}$ rational?

1. If yes we are done.
2. If no $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}\times\sqrt{2}} = \sqrt{2}^{2} = 2.$

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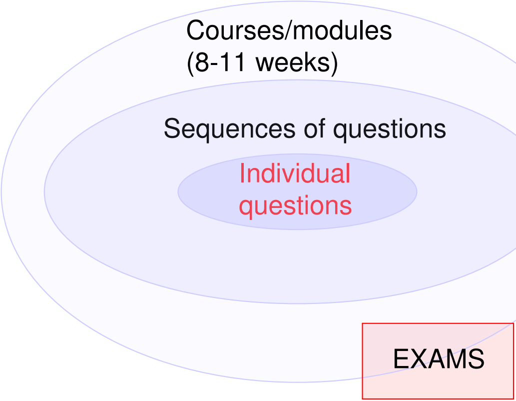
"Puzzle based learning" (for engineers).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Local classification of tasks

Knowledge
Comprehension
Application
Analysis
Synthesis
Evaluation

(Bloom 1956) *Taxonomy of educational objectives: cognitive domain*

CASIO Cassiopeia A-21S calculator (2001)

- Windows CE
- Maple V

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To what extent can existing exam questions be completed with the CAS?

Local classification

What do you have to do to answer the question?

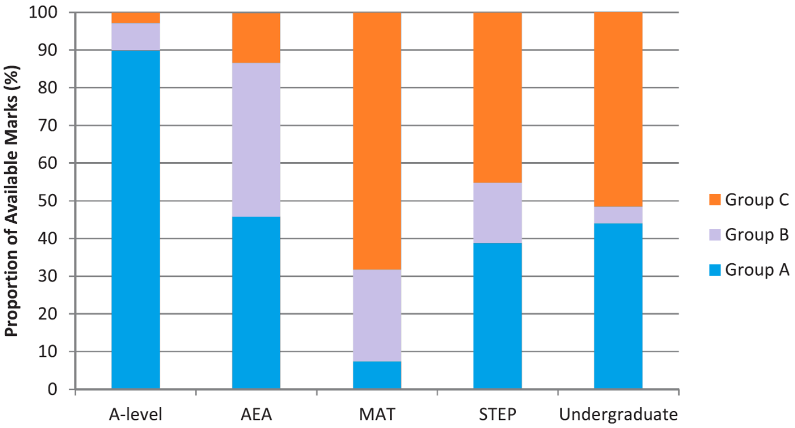
1. Factual recall
2. Carry out a routine calculation or algorithm
3. Classify some mathematical object
4. Interpret situation or answer
5. Prove, show, justify – (general argument)
6. Extend a concept
7. Criticize a fallacy
8. Construct example

Smith 1996

Group A	Group B	Group C
1. Recall factual knowledge	4. Information transfer	6. Justifying and interpreting
2. Comprehension	5. Application in new situations	7. Implications, conjectures and comparisons
3. Routine use of procedures		8. Evaluation

(Smith, *et al*, 1996)

Darlington 2015

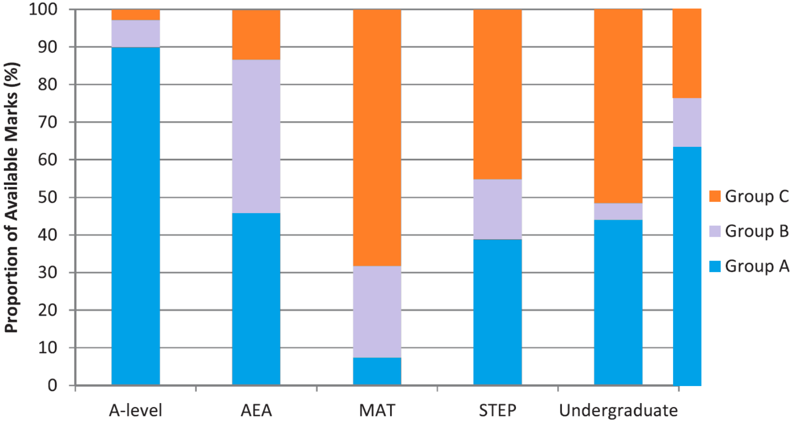


British universities offering single honours mathematics degrees [...] samples of first-year examinations of analysis and algebra.

Pointon 2002

(N=486 examination questions from year 1 university exams)

A	1. Factual recall	1.9
	2. Routine calculation	61.4
B	3. Classify some mathematical object	9.7
	4. Interpret situation or answer	2.9
C	5. Prove, show, justify	20.7
	6. Extend a concept	0.2
	7. Criticize a fallacy	0.8
	8. Construct example	2.4


Proportion of Available Marks (%)
100
90
80
70
60
50
40
30
20
10
0
A-level
AEA
MAT
STEP
Undergraduate
Group C
Group B
Group A

Routine tasks

Resolve into factors :

1. x^2+3x+2.	2. x^2+5x+6.	3. x^2+4x+3.
4. x^2-3x+2.	5. x^2-5x+6.	6. x^2-4x+3.
7. y^2+5y+4.	8. y^2+6y+8.	9. $y^2+7y+12$.
10. $y^2-9y+20$.	11. y^2-8y+7.	12. $y^2-7y+10$.
13. $z^2+8z+15$.	14. $z^2-7z+10$.	15. $z^2+9z+18$.
16. $z^2-16z+15$.	17. $z^2+13z+42$.	18. $z^2+8z+16$.
19. a^2-9a+8.	20. $a^2+10a+21$.	21. $a^2+10a+24$.
22. $a^2+9ab+14b^2$.	23. $a^2-8a+12$.	24. $a^2+11ab+24b^2$.
25. b^2-6b+9.	26. $b^2-14b+13$.	27. $b^2+11b+28$.
28. $b^2-10bc+9c^2$.	29. $b^2+9bc+8c^2$.	30. $b^2+12b+11$.
31. $x^2+16xy+63y^2$.	32. $x^2+10xy+25y^2$.	33. $x^2-14xy+24y^2$.
34. $a^2b^2-4ab+4$.	35. $a^2b^2+10ab+16$.	36. $a^2b^2+12ab+35$.
37. n^4+18n^2+65.	38. n^4-25n^2+136.	39. n^6-10n^3+25.
40. $p^2-18pq+17q^2$.	41. p^4+26p^2+69.	42. $p^2q^2-15pq+44$.

Hall & Knight, *Elementary Algebra*, (1896)

Routine tasks

Simplify

$$\frac{\frac{a-b}{a^2+ab}}{\frac{a^2-2ab+b^2}{a^4-b^4}}$$

Routine tasks

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$$= a + \frac{b^2}{a}$$

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$$= a + \frac{b^2}{a}$$

The Van Schooten Example *Principia Mathesos Universalis*. (Heller 1940)

Franciscus van Schooten (1615–1660)

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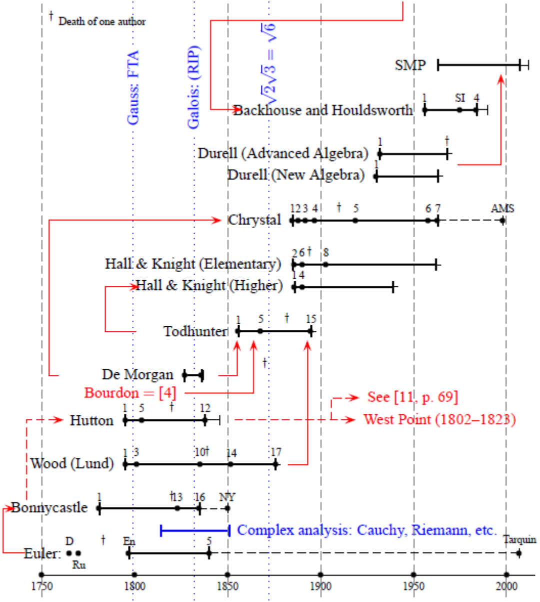
Exercises (1657) suggested Cartesian Geometry be extended to 3D.

English Algebra textbooks



Euler's algebra

A detailed study into the sources of Euler reveals that he copied most of his problems from Christoff Rudolff's Coss *which was first published in 1525 and reissued in 1553 by Michael Stifel. (Heeffer 2006)*

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No *constructive alignment*

Constructive alignment

Two basic concepts:

- Constructivist theory: Learners construct meaning from what they do.
- Alignment between planned activities and the intended learning outcomes.

(Biggs and Tang, 2011)

Routine $\rightarrow$ construct examples

A	1. Factual recall 2. Routine calculation
B	3. Classify some mathematical object 4. Interpret situation or answer
C	5. Prove, show, justify 6. Extend a concept 7. Criticize a fallacy 8. Construct example

Give me an example of...

(After John Mason)

- Please sketch a cubic.

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- Please sketch a cubic.
- Please sketch a cubic with three real roots.

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- Find a cubic which is a bijection of the reals.

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(Many situations examples can be assessed with CAA: practical)

Meta-mathematical problems

1. "Give me an example of ...."

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1. “Give me an example of ....”
2. “Give me *all* examples of ...”

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This process lies at the heart of concept formation.

Meta mathematical problems

Find a cubic with rational roots where the coordinates of the stationary points are rational.

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Nice cubics: (Johnson 2011)

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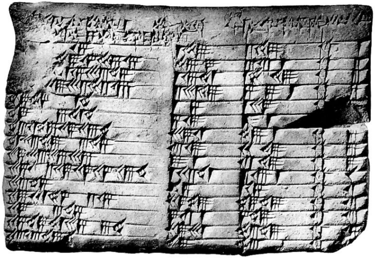
The problem is to devise the question for students.

Construct questions: Plympton 322



On balance, then, Plimpton 322 was probably (but not certainly!) a good copy of a teachers' list, with two or three columns, now missing, containing starting parameters for a set of problems, one or two columns with intermediate results (Column I and perhaps a missing column to its left), and two columns with final results (II-III).
E. Robson, *(2001)*

Generating all Pythagorean triples

Which integers which satisfy

$$a^2 + b^2 = c^2?$$

Given $m, n \in \mathbb{Z}$ with $m > n > 0$

$$a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2$$

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$$(3, 4, 5) \quad (5, 12, 13) \quad (8, 15, 17) \quad (7, 24, 25)$$

Mansfield 2017?

Available online at www.sciencedirect.com
ScienceDirect
Historia Mathematica ••• (••••) •••–•••

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HISTORIA MATHEMATICA
www.elsevier.com/locate/yhmat

Plimpton 322 is Babylonian exact sexagesimal trigonometry

Daniel F. Mansfield, N.J. Wildberger

School of Mathematics and Statistics, UNSW, Sydney, Australia

Abstract

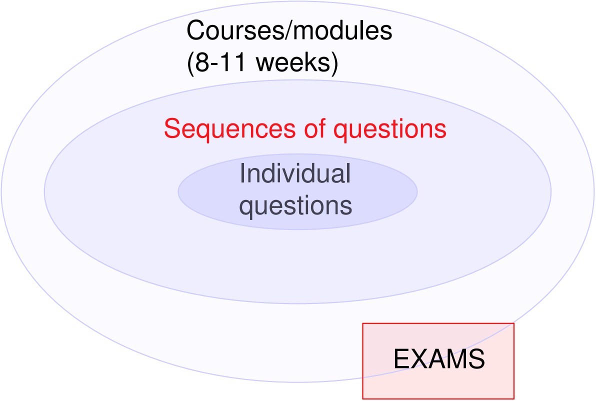
We trace the origins of trigonometry to the Old Babylonian era, between the 19th and 16th centuries B.C.E. This is well over a millennium before Hipparchus is said to have fathered the subject with his 'table of chords'. The main piece of evidence comes from the most famous of Old Babylonian tablets: Plimpton 322, which we interpret in the context of the Old Babylonian approach to triangles and their preference for numerical accuracy. By examining the evidence with this mindset, and comparing Plimpton 322 with Madhava's table of sines, we demonstrate that Plimpton 322 is a powerful, exact ratio-based trigonometric table.

"We argue that the numerical complexity of P322 proves that it is not a scribal school text, as many authors have claimed. Instead, P322 is a trigonometric table of a completely unfamiliar kind and was ahead of its time by thousands of years."
(Mansfield 2017)

Almost nice trig

For which rational angles θ are the trig formulae simple surd forms?

$$\begin{aligned}
\sin 72^\circ &= \sin(2\pi/5) = \frac{\sqrt{5+\sqrt{5}}}{2\sqrt{2}}.\\
\sin 60^\circ &= \sin(\pi/3) = \frac{\sqrt{3}}{2}.\\
\sin 45^\circ &= \sin(\pi/4) = \frac{\sqrt{2}}{2}.\\
\sin 36^\circ &= \sin(\pi/5) = \frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}.\\
\sin 30^\circ &= \sin(\pi/6) = \frac{1}{2}.\\
\sin 25.71\ldots^\circ &= \sin(\pi/7) = ?\\
\sin 22.5^\circ &= \sin(\pi/8) = \frac{\sqrt{2-\sqrt{2}}}{2}.\\
\sin 20^\circ &= \sin(\pi/9) = ?\\
\sin 18^\circ &= \sin(\pi/10) = \frac{\sqrt{5}-1}{4}.\\
\sin 16.36\ldots^\circ &= \sin(\pi/11) = ?\\
\sin 15^\circ &= \sin(\pi/12) = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}.
\end{aligned}$$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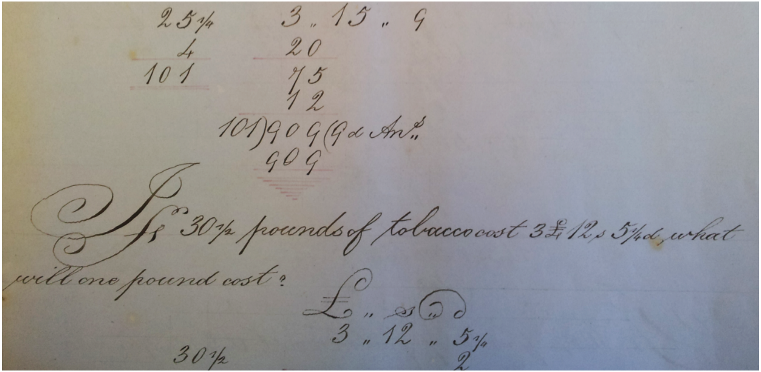
Practice is not glamorous



Joseph Phillip's copybook 1858 (age 10)

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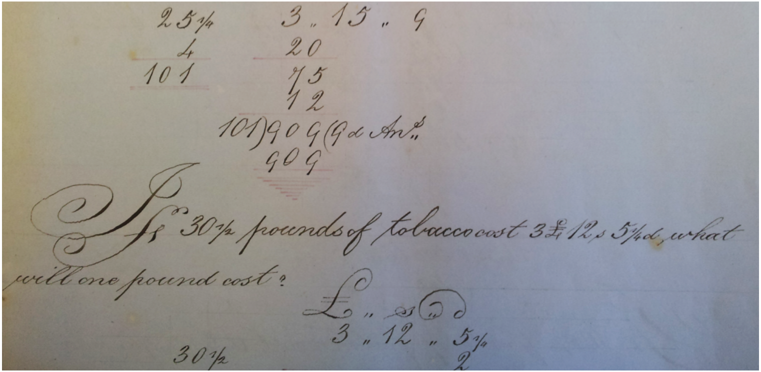

Joseph Phillip's copybook 1858 (age 10)

Regular, effortful, of limited duration, progressive not repetitive....

School Mathematics Project

We set out to create exercises where no two questions looked the same so that students were faced with new challenges all the time. This was a reaction to the Durell type texts which had long exercises of very repetitive questions.

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Quotes from Thwaites 2012

[...] the Project was based on the work of individual teachers in schools, not of university lecturers or members of committees nor self-professed "educationalists".

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Quotes from Thwaites 2012

> *[...] the Project was based on the work of individual teachers in schools, not of university lecturers or members of committees nor self-professed "educationalists".*
> *And the numbers were huge. In the first decade roughly fifty were involved in the writing and testing of text books; over two thousand had attended the teacher-training conferences; ten times as many would have used or had contact with, the SMP books in classrooms up and down the country.*

Practice: Étude

Étude:

> *a study or technical exercise, later a complete and musically intelligible composition exploring a particular technical problem in an aesthetically satisfying manner.*

(Foster 2012)

> *designing mathematical tasks that embed extensive practice of a well-defined mathematical technique within a richer, more aesthetically pleasing mathematical context.*

Progressive practice

Draw the graphs of :

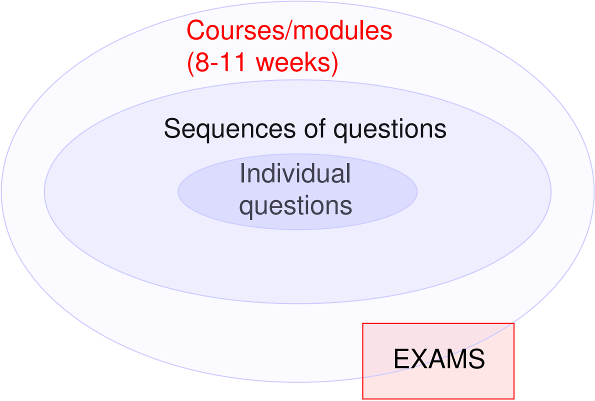
(1) $y = x^2$. (2) $y = -x^2$. (3) $y = 2x^2$.

(4) $y = x^2 + 2{\cdot}5$. (5) $y = (x-1)^2$. (6) $y = (x+2)^2 + 1$.

(7) $y = x^2 + 4x + 6$. (8) $y = x^2 - 3x + 1$.

(9) Write out a general statement of the difference between the graphs of $y = x^2$ and of $y = \pm a\{(x-b)^2 + c\}$.

Tuckey, C. O., *Examples in Algebra*, Bell & Sons, London, (1904) What is the value of a problem in isolation?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Global instruction

Many global instruction methodologies put exercises first.

- Bloom: learning for mastery
- Moore Method and related approaches
- Flipped classroom
- The books of Bob Burn

Bloom: learning for mastery

Bloom 1984: students taught by a tutor achieve test scores which are two standard deviations better than students who attend traditional classroom teaching.

Bloom: learning for mastery

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In LFM students

- are regularly tested
- are required to score 90% or
  further teaching and testing is repeated.

Bloom: learning for mastery

Bloom 1984: students taught by a tutor achieve test scores which are two standard deviations better than students who attend traditional classroom teaching.

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Now practical with online assessment.

Basis of Moore's Method

1. Mathematical problems posed to the whole class.

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Surprising consistency and stability.
Each year I ended up 40 ± 2 problems from the same place.

“Flipped classroom"

Move away from presentational lectures

1. Pre-reading

“Flipped classroom"

Move away from presentational lectures

1. Pre-reading
2. Online “reading test”

“Flipped classroom"

Move away from presentational lectures

1. Pre-reading
2. Online “reading test”
3. Lecture: audience response system,

“Flipped classroom"

Move away from presentational lectures

1. Pre-reading
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3. Lecture: audience response system,
4. Hand-in assessment

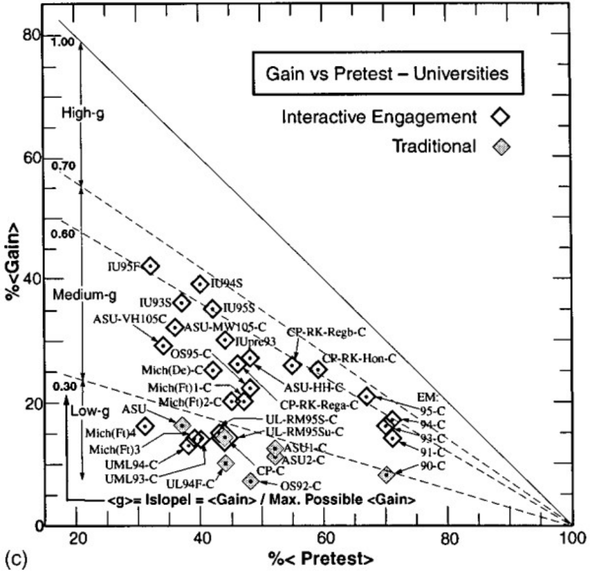
“Flipped classroom"

Move away from presentational lectures

1. Pre-reading
2. Online “reading test”
3. Lecture: audience response system,
4. Hand-in assessment
5. (Weekly workshop)

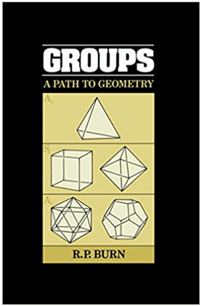
(Large groups)

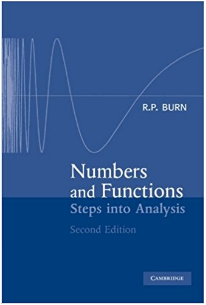
Hake 1998

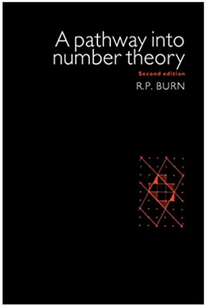


Gain vs Pretest – Universities
Interactive Engagement
Traditional
High-g
Medium-g
Low-g
1.00
0.70
0.60
0.30
%<Gain>
%< Pretest>
IU95F
IU94S
IU93S
IU95S
ASU-VH105C
ASU-MW105-C
IUpre93
CP-RK-Regb-C
OS95-C
CP-RK-Hon-C
Mich(De)-C
Mich(Ft)1-C
ASU-HH-C
Mich(Ft)2-C
CP-RK-Rega-C
EM:
95-C
94-C
93-C
91-C
90-C
ASU
UL-RM95S-C
UL-RM95Su-C
Mich(Ft)4
Mich(Ft)3
ASU1-C
ASU2-C
UML94-C
UML93-C
UL94F-C
CP-C
OS92-C
<g>= lslopel = <Gain> / Max. Possible <Gain>
(c)

The books of R. P. Burn







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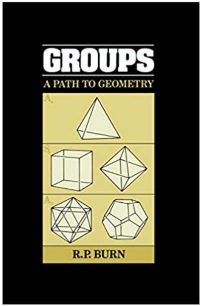

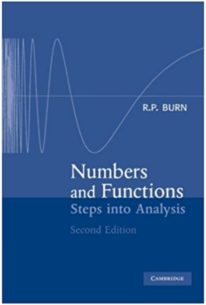

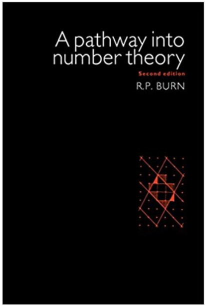

Traditional: definition; theorem; proof; example.

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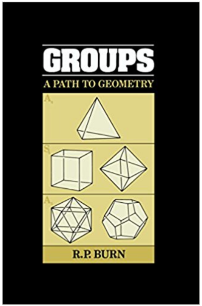

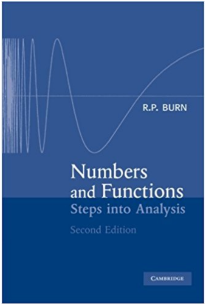

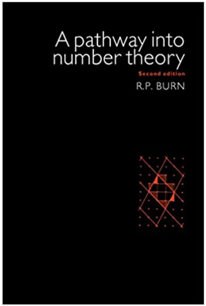

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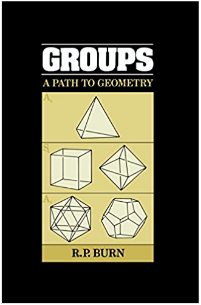

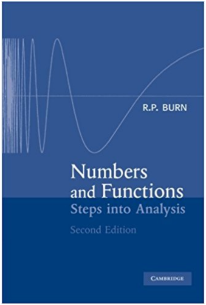

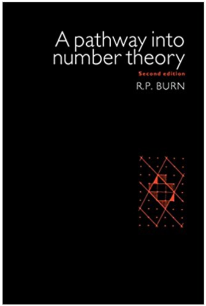

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Burn: example, proof; theorem; definition.
(Historical)

Numbers and functions

2 For $x \neq 1$, let $s_n = 1 + x + x^2 + \ldots + x^{n-1}$, a sum of only n terms.
By considering $x \cdot s_n - s_n$, prove that $s_n = (x^n - 1)/(x - 1)$.
Compare with qn 1.3(vi). It is also conventional to write s_n, as defined in the first line, in the form

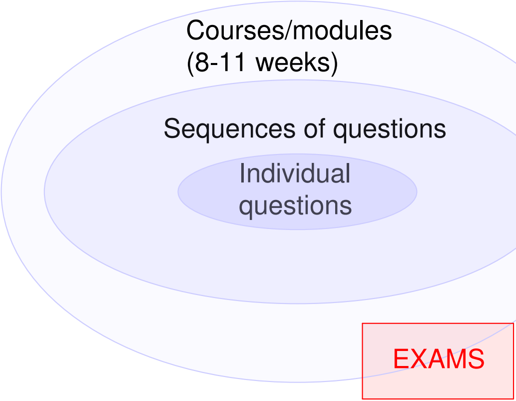
$$\sum_{r=1}^{r=n} x^{r-1} \quad \text{or} \quad \sum_{r=0}^{r=n-1} x^r.$$

3 By decomposing $1/r(r+1)$ into partial fractions, or by induction, prove that

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{n(n+1)} = 1 - \frac{1}{n+1}.$$

Express this result using the $\sum$ notation.

4 If $s_n = \sum_{r=1}^{r=n} \frac{1}{r(r+1)}$ prove that $(s_n) \to 1$ as $n \to \infty$.


Courses/modules
(8-11 weeks)
Sequences of questions
Individual
questions
EXAMS

Use and abuse (by us)

Is it possible to put an equilateral triangle onto a square grid so that all the vertices are in corners?

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> *The Mathematics Department of Moscow State University [...] was at that time actively trying to keep Jewish students (and other "undesirables") from enrolling in the department.*
> *One of the methods they used for doing this was to give the unwanted students a different set of problems on their oral exam. I was told that these problems were carefully designed to have elementary solutions (so that the Department could avoid scandals) that were nearly impossible to find.*
> *(Khovanova 2011)*

Abuse of problems. Why?

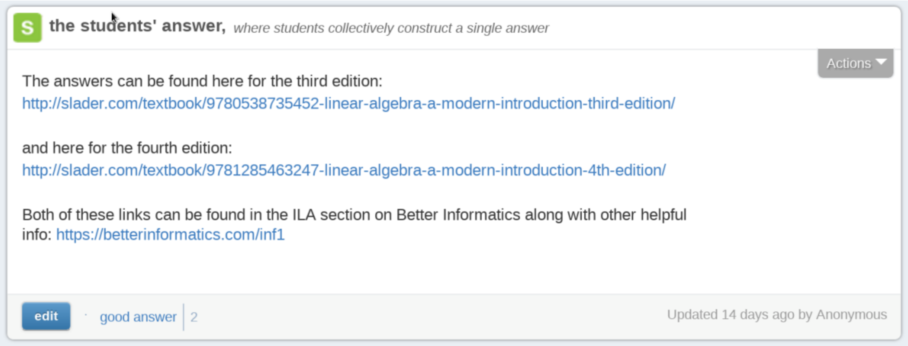
23. A steam vessel leaves Oban for Staffa with a supply of whiskey b above proof, (which is assumed to mean that $a+b$ gallons of spirit are mixed with c gallons of water) sufficient for two days' consumption provided it receive no addition to its crew. On arriving at Tobermory m of its passengers remain behind, but by reason of contrary winds its progress to Staffa the following morning is retarded, so that on its return to Oban it is with difficulty enabled to reach Iona by midnight. It here receives n additional passengers, and also p gallons of whiskey d above proof. On an average each passenger dilutes his whiskey with water till it is e below proof, and consumes q pints of the mixture daily. The vessel arrives at Oban on the evening of the third day after its departure, by which time the supplies of whiskey are both exhausted. Required the number of passengers on board when it left Oban, and the number of gallons of whiskey in the first supply. (*Comp.* p. 178.)

$$(1)\ \text{Ans.}\ 2m-n+\frac{8p}{q}\cdot\frac{a+d}{a-e}\cdot\frac{a+e-c}{a+d+c}.$$

$$(2)\ \text{Ans.}\ (2m-n)\frac{q}{4}\cdot\frac{a-e}{a+b}\cdot\frac{a+b+c}{a-e+c}+p\cdot\frac{a+d}{a+d+c}.$$

(Wood/Lund 1876)

Use and abuse (by students)



See also `https://www.coursehero.com/`

Competitive situations and “play"

- Mathematical competitions (e.g. Olympiad)

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  Carse*, Finite and Infinite Games (1986)*

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Malcolm Swan: tests worth teaching to....

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  They have history and cultural significance.
- Familiarity and fashion.
  What do we do about the internet?!