Hound and Hare

A hound starts in pursuit of a hare at a distance of 30 of his own leaps from her. He takes 5 leaps while she takes 6 but covers as much ground in 2 as she in 3. In how many leaps of each will the hare be caught?
Mathematical tasks: past, present and future

Chris Sangwin

School of Mathematics
University of Edinburgh

October 2017
Outline

Courses/modules (8-11 weeks)

Sequences of questions

Individual questions

EXAMS
Hound and Hare

A hound starts in pursuit of a hare at a distance of 30 of his own leaps from her. He takes 5 leaps while she takes 6 but covers as much ground in 2 as she in 3. In how many leaps of each will the hare be caught?
Hound and Hare

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Hound and the hare
Hound and the hare

- Alcuin of York *Problems to Sharpen the Youth*. (790CE)
  (A collection of 247 problems from 100CE)
The problem as artefact

- Problems have a history and cultural significance.
The problem as artefact

- Problems have a history and cultural significance. We attribute theorems, but not problems.
The problem as artefact

- Problems have a history and cultural significance. We attribute theorems, but not problems.
- Puzzles are enduringly popular.
The problem as artefact

- Problems have a history and cultural significance. We attribute theorems, but not problems.
- Puzzles are enduringly popular. Scale from “trivial” to research.
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How do we design & use them, and to what effect?
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- How do we design & use them, and to what effect?
- Familiarity and fashion. What do we do about the internet?!
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- Availability E.g. “concept inventories”, research instruments are hidden.
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- Familiarity and fashion. What do we do about the internet?!
- Availability
  E.g. “concept inventories”, research instruments are hidden.

Problems as “mathematical verse”.

Chris Sangwin (University of Edinburgh)
Well-versed

A phrase meaning

*highly experienced, practiced, or skilled; very knowledgeable; learned.*
Patterns of thought

Alice looks at Bob and Bob looks at Clare. Alice is married but Clare is not. Prove that a married person looks at an unmarried person.
Patterns of thought

Alice looks at Bob and Bob looks at Clare. Alice is married but Clare is not. Prove that a married person looks at an unmarried person.

\[ A(M) \rightarrow B(?) \rightarrow C(U). \quad (M) \rightarrow (U) \]
Patterns of thought

Alice looks at Bob and Bob looks at Clare. Alice is married but Clare is not. Prove that a married person looks at an unmarried person.

\[ A(M) \rightarrow B(?) \rightarrow C(U). \quad (M) \rightarrow (U) \]

Prove that an irrational power of an irrational number can be rational. \(\sqrt{2}\) is irrational. Consider \(\sqrt{2}^{\sqrt{2}}\).
Patterns of thought

Alice looks at Bob and Bob looks at Clare.
Alice is married but Clare is not.
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Prove that an irrational power of an irrational number can be rational.

\( \sqrt{2} \) is irrational. Consider \( \sqrt{2}^{\sqrt{2}} \).

\( \sqrt{2}^{\sqrt{2}} \) rational?

1. If yes we are done.

2. If no \( \left( \sqrt{2}^{\sqrt{2}} \right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2} \times \sqrt{2}} = \sqrt{2}^{2} = 2. \)
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“Puzzle based learning” (for engineers).
Courses/modules (8-11 weeks)

Sequences of questions

Individual questions

EXAMS
Local classification of tasks

(Bloom 1956) *Taxonomy of educational objectives: cognitive domain*

<table>
<thead>
<tr>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge</td>
</tr>
<tr>
<td>Comprehension</td>
</tr>
<tr>
<td>Application</td>
</tr>
<tr>
<td>Analysis</td>
</tr>
<tr>
<td>Synthesis</td>
</tr>
<tr>
<td>Evaluation</td>
</tr>
</tbody>
</table>
CASIO Cassiopeia A-21S calculator (2001)

- Windows CE
- Maple V
CASIO Cassiopeia A-21S calculator (2001)

- Windows CE
- Maple V

To what extent can existing exam questions be completed with the CAS?
Local classification

What do you have to do to answer the question?

1. Factual recall
2. Carry out a routine calculation or algorithm
3. Classify some mathematical object
4. Interpret situation or answer
5. Prove, show, justify – (general argument)
6. Extend a concept
7. Criticize a fallacy
8. Construct example
<table>
<thead>
<tr>
<th>Group A</th>
<th>Group B</th>
<th>Group C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Recall factual knowledge</td>
<td>4. Information transfer</td>
<td>6. Justifying and interpreting</td>
</tr>
<tr>
<td>2. Comprehension</td>
<td>5. Application in new situations</td>
<td>7. Implications, conjectures and comparisons</td>
</tr>
<tr>
<td>3. Routine use of procedures</td>
<td></td>
<td>8. Evaluation</td>
</tr>
</tbody>
</table>

(Smith, et al, 1996)
British universities offering single honours mathematics degrees [...] samples of first-year examinations of analysis and algebra.
Pointon 2002

(N=486 examination questions from year 1 university exams)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>1. Factual recall</td>
</tr>
<tr>
<td></td>
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</tr>
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</tr>
<tr>
<td></td>
<td>8. Construct example</td>
</tr>
</tbody>
</table>
Routine tasks

Resolve into factors:

1. \(x^2 + 3x + 2\).
2. \(x^2 + 5x + 6\).
3. \(x^2 + 4x + 3\).
4. \(x^2 - 3x + 2\).
5. \(x^2 - 5x + 6\).
6. \(x^2 - 4x + 3\).
7. \(y^2 + 5y + 4\).
8. \(y^2 + 6y + 8\).
9. \(y^2 + 7y + 12\).
10. \(y^2 - 9y + 20\).
11. \(y^2 - 8y + 7\).
12. \(y^2 - 7y + 10\).
13. \(z^2 + 8z + 15\).
14. \(z^2 - 7z + 10\).
15. \(z^2 + 9z + 18\).
16. \(z^2 - 16z + 15\).
17. \(z^2 + 13z + 42\).
18. \(z^2 + 8z + 16\).
19. \(a^2 - 9a + 8\).
20. \(a^2 + 10a + 21\).
21. \(a^2 + 10a + 24\).
22. \(a^2 + 9ab + 14b^2\).
23. \(a^2 - 8a + 12\).
24. \(a^2 + 11ab + 24b^2\).
25. \(b^2 - 6b + 9\).
26. \(b^2 - 14b + 13\).
27. \(b^2 + 11b + 28\).
28. \(b^2 - 10bc + 9c^2\).
29. \(b^2 + 9bc + 8c^2\).
30. \(b^2 + 12b + 11\).
31. \(x^2 + 16xy + 63y^2\).
32. \(x^2 + 10xy + 25y^2\).
33. \(x^2 - 14xy + 24y^2\).
34. \(a^2b^2 - 4ab + 4\).
35. \(a^2b^2 + 10ab + 16\).
36. \(a^2b^2 + 12ab + 35\).
37. \(n^4 + 18n^2 + 65\).
38. \(n^4 - 25n^2 + 136\).
39. \(n^6 - 10n^3 + 25\).
40. \(p^2 - 18pq + 17q^2\).
41. \(p^4 + 26p^2 + 69\).
42. \(p^2q^2 - 15pq + 44\).

Hall & Knight, *Elementary Algebra*, (1896)
Routine tasks

Simplify

\[
\frac{a-b}{a^2+ab} \div \frac{a^2-2ab+b^2}{a^4-b^4}
\]
Routine tasks

Simplify

\[
\frac{a-b}{a^2+ab} \frac{a^2-2ab+b^2}{a^4-b^4} = a + \frac{b^2}{a}
\]
Routine tasks

Simplify

\[
\frac{a - b}{a^2 + ab} \frac{a^2 - 2ab + b^2}{a^4 - b^4} = a + \frac{b^2}{a}
\]

The Van Schooten Example *Principia Mathesos Universalis*. (Heller 1940)
Franciscus van Schooten (1615–1660)
Franciscus van Schooten (1615–1660)

Exercises (1657) suggested Cartesian Geometry be extended to 3D.
English Algebra textbooks

- Gauss: FTA
- Galois: (RIP)
- Chrystal
- Durell (Advanced Algebra)
- Durell (New Algebra)
- Hall & Knight (Elementary)
- Hall & Knight (Higher)
- Todhunter
- De Morgan
- Bourdon = [4]
- Hutton
- Wood (Lund)
- Bonnycastle
- Euler: D
- En
- Complex analysis: Cauchy, Riemann, etc.

- SMP
- SI
- AMS
- See [11, p. 69]
- West Point (1802–1823)

Death of one author
1. \sqrt{2} = \sqrt{6}

1750 1800 1850 1900 1950 2000
Euler’s algebra

A detailed study into the sources of Euler reveals that he copied most of his problems from Christoff Rudolff’s Coss which was first published in 1525 and reissued in 1553 by Michael Stifel. (Heffer 2006)
Euler’s algebra

A detailed study into the sources of Euler reveals that he copied most of his problems from Christoff Rudolff’s Coss which was first published in 1525 and reissued in 1553 by Michael Stifel. (Heeffer 2006)

No constructive alignment
Constructive alignment

Two basic concepts:

- Constructivist theory: Learners construct meaning from what they do.
- Alignment between planned activities and the intended learning outcomes.

(Biggs and Tang, 2011)
### Routine $\rightarrow$ construct examples

| A   | 1. Factual recall  
|     | 2. **Routine calculation**  
| B   | 3. Classify some mathematical object  
|     | 4. Interpret situation or answer  
| C   | 5. Prove, show, justify  
|     | 6. Extend a concept  
|     | 7. Criticize a fallacy  
|     | 8. **Construct example**  

Give me an example of...

(After John Mason)

- Please sketch a cubic.
Give me an example of...

(After John Mason)
- Please sketch a cubic.
- Please sketch a cubic with three real roots.
Give me an example of...

(After John Mason)

- Please sketch a cubic.
- Please sketch a cubic with three real roots.
- Please sketch a cubic with one negative root and two positive real roots.
Give me an example of...

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- ...

...
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- ...
- Find a cubic which is a bijection of the reals.
Give me an example of...

(After John Mason)
- Please sketch a cubic.
- Please sketch a cubic with three real roots.
- Please sketch a cubic with one negative root and two positive real roots.
- ...
- Find a cubic which is a bijection of the reals.

(Many situations examples can be assessed with CAA: practical)
“Give me an example of ....”
Meta-mathematical problems

1. “Give me an example of ....”
2. “Give me all examples of ...”
Meta-mathematical problems

1. “Give me an example of ....”

2. “Give me all examples of ...”

3. Let $X$ be the set which which ...
Meta-mathematical problems

1. “Give me an example of ....”
2. “Give me all examples of ...”
3. *Let X be the set which which ...*

This process lies at the heart of concept formation.
Find a cubic with rational roots where the coordinates of the stationary points are rational.
Meta mathematical problems

Find a cubic with rational roots where the coordinates of the stationary points are rational.
Nice cubics: (Johnson 2011)
Meta mathematical problems

Find a cubic with rational roots where the coordinates of the stationary points are rational.
Nice cubics: (Johnson 2011)

The problem is to devise the question for students.
On balance, then, Plimpton 322 was probably (but not certainly!) a good copy of a teachers’ list, with two or three columns, now missing, containing starting parameters for a set of problems, one or two columns with intermediate results (Column I and perhaps a missing column to its left), and two columns with final results (II-III).

E. Robson, (2001)
Generating all Pythagorean triples

Which integers which satisfy

\[ a^2 + b^2 = c^2? \]

Given \( m, n \in \mathbb{Z} \) with \( m > n > 0 \)

\[ a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2 \]
Generating all Pythagorean triples

Which integers which satisfy

\[ a^2 + b^2 = c^2? \]

Given \( m, n \in \mathbb{Z} \) with \( m > n > 0 \)

\[ a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2 \]

\[ (3, 4, 5) \quad (5, 12, 13) \quad (8, 15, 17) \quad (7, 24, 25) \]
“We argue that the numerical complexity of P322 proves that it is not a scribal school text, as many authors have claimed. Instead, P322 is a trigonometric table of a completely unfamiliar kind and was ahead of its time by thousands of years.”
(Mansfield 2017)
Almost nice trig

For which rational angles \( \theta \) are the trig formulae simple surd forms?

\[
\begin{align*}
\sin 72^\circ &= \sin(2\pi / 5) = \frac{\sqrt{5} + \sqrt{5}}{2\sqrt{2}}. \\
\sin 60^\circ &= \sin(\pi / 3) = \frac{\sqrt{3}}{2}. \\
\sin 45^\circ &= \sin(\pi / 4) = \frac{\sqrt{2}}{2}. \\
\sin 36^\circ &= \sin(\pi / 5) = \frac{\sqrt{5} - \sqrt{5}}{2\sqrt{2}}. \\
\sin 30^\circ &= \sin(\pi / 6) = \frac{1}{2}. \\
\sin 25.71 \ldots ^\circ &= \sin(\pi / 7) = ? \\
\sin 22.5^\circ &= \sin(\pi / 8) = \frac{\sqrt{2} - \sqrt{2}}{2}. \\
\sin 20^\circ &= \sin(\pi / 9) = ? \\
\sin 18^\circ &= \sin(\pi / 10) = \frac{\sqrt{5} - 1}{4}. \\
\sin 16.36 \ldots ^\circ &= \sin(\pi / 11) = ?
\sin 15^\circ &= \sin(\pi / 12) = \frac{\sqrt{3} - 1}{2\sqrt{2}} = \frac{\sqrt{2} - \sqrt{3}}{2}.
\end{align*}
\]
Courses/modules
(8-11 weeks)

Sequences of questions

Individual
questions

EXAMS
Practice is not glamorous

Joseph Phillip's copybook 1858 (age 10)
Practice is not glamorous

Joseph Phillip's copybook 1858 (age 10)

Regular, effortful, of limited duration, progressive not repetitive....
School Mathematics Project

*We set out to create exercises where no two questions looked the same so that students were faced with new challenges all the time. This was a reaction to the Durell type texts which had long exercises of very repetitive questions.*
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Quotes from Thwaites 2012

[…] the Project was based on the work of individual teachers in schools, not of university lecturers or members of committees nor self-professed “educationalists”.
We set out to create exercises where no two questions looked the same so that students were faced with new challenges all the time. This was a reaction to the Durell type texts which had long exercises of very repetitive questions.

Quotes from Thwaites 2012

[…] the Project was based on the work of individual teachers in schools, not of university lecturers or members of committees nor self-professed “educationalists”. And the numbers were huge. In the first decade roughly fifty were involved in the writing and testing of text books; over two thousand had attended the teacher-training conferences; ten times as many would have used or had contact with, the SMP books in classrooms up and down the country.
Practice: Étude

Étude:

*a study or technical exercise, later a complete and musically intelligible composition exploring a particular technical problem in an aesthetically satisfying manner.*

(Foster 2012)

*designing mathematical tasks that embed extensive practice of a well-defined mathematical technique within a richer, more aesthetically pleasing mathematical context.*
Draw the graphs of:

1. \( y = x^2 \).
2. \( y = -x^2 \).
3. \( y = 2x^2 \).
4. \( y = x^2 + 2.5 \).
5. \( y = (x - 1)^2 \).
6. \( y = (x + 2)^2 + 1 \).
7. \( y = x^2 + 4x + 6 \).
8. \( y = x^2 - 3x + 1 \).
9. Write out a general statement of the difference between the graphs of \( y = x^2 \) and of \( y = \pm a \{ (x - b)^2 + c \} \).

What is the value of a problem in isolation?
Courses/modules (8-11 weeks)

Sequences of questions

Individual questions

EXAMS
Global instruction

Many global instruction methodologies put exercises first.

- Bloom: learning for mastery
- Moore Method and related approaches
- Flipped classroom
- The books of Bob Burn
Bloom 1984: students taught by a tutor achieve test scores which are two standard deviations better than students who attend traditional classroom teaching.
Bloom: learning for mastery

Bloom 1984: students taught by a tutor achieve test scores which are two standard deviations better than students who attend traditional classroom teaching.

In LFM students

- are regularly tested
- are required to score 90% or further teaching and testing is repeated.
Bloom 1984: students taught by a tutor achieve test scores which are two standard deviations better than students who attend traditional classroom teaching.

In LFM students

- are regularly tested
- are required to score 90% or further teaching and testing is repeated.

Now practical with online assessment.
Basis of Moore’s Method

1. Mathematical problems posed to the whole class.
Basis of Moore’s Method

1. Mathematical problems posed to the whole class.
2. Students solve problems independently.
Basis of Moore’s Method

1. Mathematical problems posed to the whole class.
2. Students solve problems independently.
3. Students present their solutions to the class.

Surprising consistency and stability. Each year I ended up 40 ± 2 problems from the same place.
Basis of Moore’s Method

1. Mathematical problems posed to the whole class.
2. Students solve problems independently.
3. Students present their solutions to the class.
4. Students discuss solutions.

Surprising consistency and stability. Each year I ended up 40 ± 2 problems from the same place.

Chris Sangwin (University of Edinburgh)
Basis of Moore’s Method

1. Mathematical problems posed to the whole class.
2. Students solve problems independently.
3. Students present their solutions to the class.
4. Students discuss solutions.
5. Students decide if answers are correct.

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Surprising consistency and stability.
Each year I ended up $40 \pm 2$ problems from the same place.
“Flipped classroom"

Move away from presentational lectures

- Pre-reading
“Flipped classroom”

Move away from presentational lectures

1. Pre-reading
2. Online “reading test”
“Flipped classroom”

Move away from presentational lectures

1. Pre-reading
2. Online “reading test”
3. Lecture: audience response system,
“Flipped classroom"

Move away from presentational lectures

1. Pre-reading
2. Online “reading test”
3. Lecture: audience response system,
4. Hand-in assessment
"Flipped classroom"

Move away from presentational lectures

1. Pre-reading
2. Online “reading test”
3. Lecture: audience response system,
4. Hand-in assessment
5. (Weekly workshop)

(Large groups)
The books of R. P. Burn
The books of R. P. Burn

Traditional: definition; theorem; proof; example.
The books of R. P. Burn

Traditional: definition; theorem; proof; example.

Burn: example, proof; theorem; definition.
The books of R. P. Burn

Traditional: definition; theorem; proof; example.

Burn: example, proof; theorem; definition. (Historical)
Numbers and functions

5 Series: infinite sums

2 For $x \neq 1$, let $s_n = 1 + x + x^2 + \ldots + x^{n-1}$, a sum of only $n$ terms.
By considering $x \cdot s_n - s_n$, prove that $s_n = (x^n - 1)/(x - 1)$.
Compare with qn 1.3(vi). It is also conventional to write $s_n$, as defined in the first line, in the form
\[
\sum_{r=1}^{r=n} x^{r-1} \quad \text{or} \quad \sum_{r=0}^{r=n-1} x^r.
\]

3 By decomposing $1/r(r + 1)$ into partial fractions, or by induction, prove that
\[
\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n(n + 1)} = 1 - \frac{1}{n + 1}.
\]
Express this result using the $\sum$ notation.

4 If $s_n = \sum_{r=1}^{r=n} \frac{1}{r(r + 1)}$ prove that $(s_n) \to 1$ as $n \to \infty$. 
Courses/modules
(8-11 weeks)

Sequences of questions

Individual questions

EXAMS
Use and abuse (by us)

Is it possible to put an equilateral triangle onto a square grid so that all the vertices are in corners?
Use and abuse (by us)

Is it possible to put an equilateral triangle onto a square grid so that all the vertices are in corners?

The Mathematics Department of Moscow State University [...] was at that time actively trying to keep Jewish students (and other “undesirables”) from enrolling in the department.
Is it possible to put an equilateral triangle onto a square grid so that all the vertices are in corners?

The Mathematics Department of Moscow State University [...] was at that time actively trying to keep Jewish students (and other “undesirables”) from enrolling in the department. One of the methods they used for doing this was to give the unwanted students a different set of problems on their oral exam. I was told that these problems were carefully designed to have elementary solutions (so that the Department could avoid scandals) that were nearly impossible to find. (Khovanova 2011)
23. A steam vessel leaves Oban for Staffa with a supply of whiskey \( b \) above proof, (which is assumed to mean that \( a+b \) gallons of spirit are mixed with \( c \) gallons of water) sufficient for two days’ consumption provided it receive no addition to its crew. On arriving at Tobermory \( m \) of its passengers remain behind, but by reason of contrary winds its progress to Staffa the following morning is retarded, so that on its return to Oban it is with difficulty enabled to reach Iona by midnight. It here receives \( n \) additional passengers, and also \( p \) gallons of whiskey \( d \) above proof. On an average each passenger dilutes his whiskey with water till it is \( e \) below proof, and consumes \( q \) pints of the mixture daily. The vessel arrives at Oban on the evening of the third day after its departure, by which time the supplies of whiskey are both exhausted. Required the number of passengers on board when it left Oban, and the number of gallons of whiskey in the first supply. (Comp. p. 178.)

(1) Ans. \( 2m-n+\frac{8p}{q} \cdot \frac{a+d}{a-e} \cdot \frac{a+e-c}{a+d+c} \).

(2) Ans. \( (2m-n)q \cdot \frac{a-e}{4} \cdot \frac{a+b+c}{a-e+c} + p \cdot \frac{a+d}{a+d+c} \).

(Wood/Lund 1876)
Use and abuse (by students)

The answers can be found here for the third edition:

and here for the fourth edition:

Both of these links can be found in the ILA section on Better Informatics along with other helpful info: https://betterinformatics.com/inf1

See also https://www.coursehero.com/
Competitive situations and “play"

- Mathematical competitions (e.g. Olympiad)

Carse, Finite and Infinite Games (1986)

Malcolm Swan: tests worth teaching to...
Competitive situations and “play"

- Mathematical competitions (e.g. Olympiad)
- Societal competitions (e.g. exams)

To be trained is to be prepared against surprise.
To be educated is to be prepared for surprise.

Carse, Finite and Infinite Games (1986)
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Malcolm Swan: tests worth teaching to....
Writing problems is a significant intellectual challenge.
Conclusion

- Writing problems is a significant intellectual challenge.
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