MatRIC Centre for Research, Innovation and Coordination of Mathematics Teaching

## Mathematical Modelling Symposium

# Didactic aspects of teaching modelling to undergraduate students

Paul Hernandez-Martinez
Pauline Vos
Barbara Jaworski



Loughborough University

# Didactics of Mathematics

- Didactics' is about the transformation of disciplinary knowledge into forms through which learners can develop their own versions of that knowledge. We want students to learn mathematics, which is not easily accessible in the world around us.
- Students need to be offered tasks and activity through which to appreciate mathematics and develop mathematical concepts.
- The process of transforming mathematics into such opportunities for students is a didactical process which is the practical task of the teacher of mathematics.
(Jaworski & Huang, 2014)

Students need to be offered tasks and activities through which to appreciate mathematics, learn about its usefulness, and develop mathematical concepts, habits and problem solving skills, gain confidence and flexibility in approaches.

How can we, as teachers, achieve this?

# For the teacher of mathematics …

If we are teaching mathematics, then didactics is about the translation of the concepts, habits, and problem solving skills of mathematics into aims, curricula, materials and (sequences of) activities for the learners of mathematics.

How do we see ‘modelling’ to fit with this?



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# Mathematical modelling

Mathematical Modelling can be seen either

(a) as a vehicle for the learning of mathematics, or

(b) as something to be learned in its own right, perhaps in order to apply it in scientific or engineering contexts

(Julie & Mudaly, 2007)



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# Three presentations

Paul Hernandez-Martinez, Lougborough University, UK

Collaborative learning in mathematical modelling tasks

Pauline Vos, UiA, Norway

“Authenticity" in the teaching and learning of mathematical modelling

Barbara Jaworski, Lougborough University, UK

Modelling and inquiry – how are they related?

***Discussion***

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## Mathematical Modelling Symposium

# Modelling and Inquiry – how are they related?

Barbara Jaworski



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# Inquiry-based teaching and learning

- Seeking to enable students to develop a more conceptually  based understanding of mathematics
- Use of inquiry approaches in three layers
- The ESUM Project – an innovation in teaching mathematics to first year engineering students.
- Developmental research



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# Theoretical base

We see mathematics knowledge growing in social settings through mediational processes and the use of tools such as inquiry-based tasks and approaches to teaching

(Schmittau, 2003; Wertsch, 1991).

# Nested Layers of Inquiry

The developmental methodology involving nested layers of inquiry (A, B & C with A□B□C) with students’ learning of mathematics at the centre:

A *Inquiry in mathematics*

- involves students in learning and understanding mathematics through inquiry.

*B Inquiry in developing mathematics teaching*

- involves questioning teaching approaches and the design of teaching, to understand the basis of teaching decisions and ways of improving teaching for better learning outcomes.

C Inquiry in the research process

- inspects the other two layers to gain insights to the developmental processes in both layers, and their outcomes (Jaworski, 2006)

# The ESUM Project

The ESUM study employed a developmental methodology, incorporating an inquiry-based approach, in which research both studied developmental practice and contributed to development

(Jaworski, 2003)



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# Creating mathematical meaning

- RQ1: When a (developmental research) project seeks to enhance students' meaning making of mathematics, how can we gain insights to students' mathematical meanings?
- RQ2: How can we characterise mathematical meaning-making in ways which aid its creation?
  In what ways can the SEFI/Niss competence framework aid characterisation?
  (SEFI-2013 http://sefi.htw-aalen.de/).

Jaworski, 2013 (CERME Paper)



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# KOM Competences

| *The ability to ask and answer questions in and with mathematics* | *The ability to deal with mathematical language and tools* |
|---|---|
| 1. Thinking mathematically<br>2. Reasoning mathematically<br>3. Posing and solving mathematical problems<br>4. Modelling mathematically | 5. Representing mathematical entities<br>6. Handling mathematical symbols and formalism<br>7. Communicating in, with and about mathematics<br>8. Making use of aids and tools |



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# *Three dimensions …*

… for specifying and measuring progress in learning with respect to competency:

- *Degree of coverage*: The extent to which the person masters the characteristic aspects of a competency;
- *Radius of action*: The contexts and situations in which a person can activate a competency and
- *Technical level*: How conceptually and technically advanced the entities and tools are with which the person can activate the competence.

How to address these dimensions is an issue to consider.

| | Task -- Inquiry-based | Competences |
|---|---|---|
| 1 | Think about what we mean by a function and write down two examples. Try to make them different examples. | 1, 2, 5, 6, 7 |
| 2 | In the topic area of real valued functions of one variable | |
| a) | Consider the function $f(x) = x^2 + 2x$ (x is real)<br>Give an equation of a line that intersects the graph of this function (i) Twice (ii) Once (iii) Never | 1, 2, 3, 5, 6, 7 |
| b) | If we have the function $f(x) = ax^2 + bx + c$.<br>What can you say about lines which intersect this function twice? | 1, 2, 7, (8) |
| c) | Write down equations for three straight lines and draw them in GeoGebra | 1, 2, 5, 6, 7, 8 |
| d) | Find a (quadratic) function such that the graph of the function cuts one of your lines twice, one of them only once, and the third not at all and show the result in GeoGebra. | 1, 2, 3, 7, 8 |
| e) | Repeat for three different lines (what does it mean to be different?) | 1, 2, 3, 5, 6, 7, (8) |



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# Some questions

- What makes these tasks "inquiry-based" tasks?
- Where is the "modelling" competence?
- What does modelling offer towards achieving students (conceptual) understanding of mathematics that takes us *beyond* inquiry?
- Do we want students to learn mathematics, or to learn modelling, or both?

Encouraging exploration/investigation requiring engagement & collaboration
Asking own questions
Developing an argument



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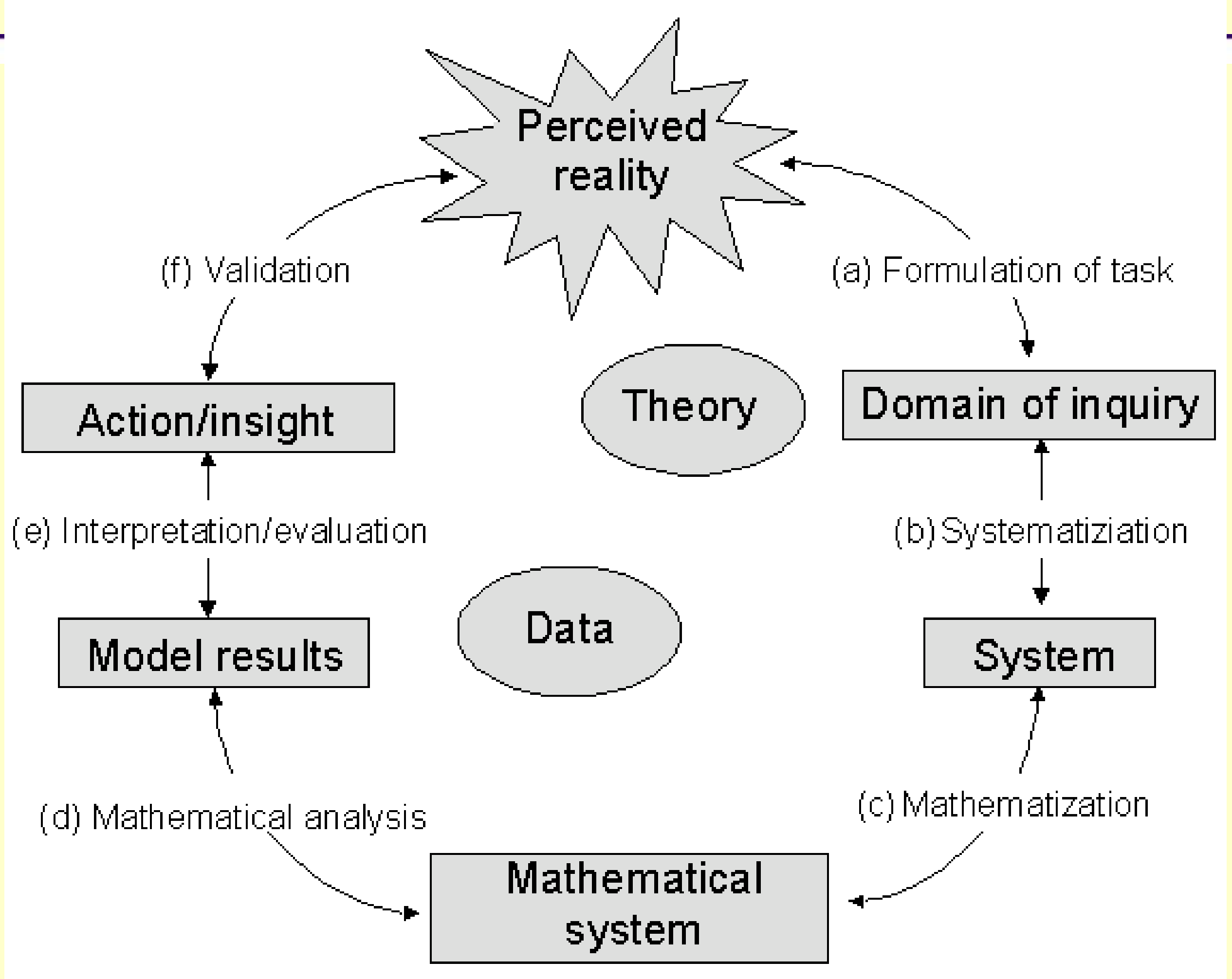
# A Modelling Cycle




**A visual representation of the mathematical modelling process (adapted from Blomhøj & Jensen, 2003). (Blomhøj & Jensen, 2007).**



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# Three dimensions for modelling

1. … describing the *degree of coverage*, meaning which parts of the modelling process the students are working with and at what level of reflection,
2. … to do with *the technical level* of the students activities involved in the modelling process, meaning what kind of mathematics they use and how flexible they do it, and
3. … to do with variation in the types of situations and contexts in which the students can actually activate their mathematical modelling competency, in short called the *radius of action.*



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# Modelling tasks

- How can progress in mathematical modelling competency be described in ways that support the development of good and coherent teaching practices at different educational levels?
- (Blomhøj & Jensen, 2007).

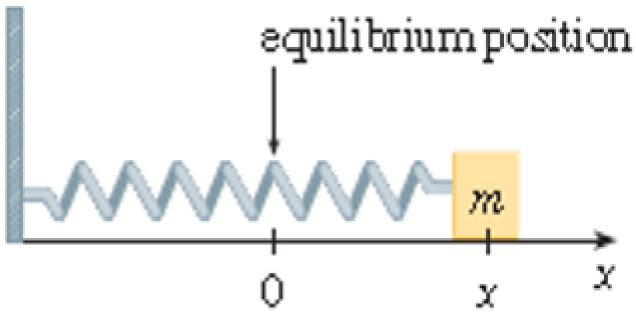
# Task 1

- A car driving 60 km/h passes a car driving at a speed of 50 km/h. When the cars are right beside each other a girl appears some meters ahead. The drivers react in the same way and the cars have brakes of equal quality. The car with 50 km/h stops right in front of the girl, while the other car, with the initial speed of 60 km/h, hits the girl with 44 km/h.   Seven out of ten die in such an accident.
- Given this text the students are simply asked: Can this be true?

# Task 2

There is an object of mass $m$ attached to the end of a spring as in the illustration. **Hooke's law** states that the force exerted over a spring that is stretched $x$ units from its resting position (equilibrium) is proportional to the distance $x$.

Use Newton's second law of motion (Force is equal to mass times acceleration) to model this type of motion (known as simple harmonic motion) with a second order differential equation. Then, solve the equation.




Now suppose that a spring with a 3 kg mass is held stretched 0.6 m beyond its equilibrium position by a force of 20 N. If the spring begins at its equilibrium position (when $t$=0 what is $x$?) but a push gives it an initial velocity of 1.2 m/s (when $t$=0 what is the velocity?), find the position of the mass after $t$ seconds.

# Task 3

A company wants to make boxes that hold $0.02m^3$ of nuts and bolts. It should have a square base and double thickness on top and bottom. The cardboard costs £0.30 per $m^2$. What size of box is the most economical?



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# Task 4

William Gosset worked at the Guinness brewery in Dublin and wanted to try to understand quality control.  He wanted to help Guinness make better beer and he needed a way that would allow him to compare small samples from different types of barley to see if there was any difference in the quality of the beer produced.

He wanted to collect data from small samples of beer, made with different types of barley, and find some way of comparing these samples systematically so that he could extrapolate to the whole of the population.

In order to achieve this he needed a mathematical model and the model he produced, we call the T-test.



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# Task 5

A paint factory uses two machines for the production of their red colour. They took a sample of nine from each machine and measured the brightness of the colour.

Is there any evidence that the choice of the machine affects the brightness of the red colour on a 5% level of significance?

# How are these tasks the same or different?

- The ESUM inquiry-based tasks remained within the world of mathematics.
- Students engaged in inquiry which drew them into the mathematics, tackling questions through exploration and investigation, and using an electronic environment to aid their inquiry.
- The ‘modelling’ tasks all have one feature different from the inquiry-based tasks -- they are set in some context: real world or pseudo-real-world. The context is used for a variety of purposes.

# The (didactical) purpose of 'context'

Task 1 – The two cars and the accident

- Students are asked to engage with real-world aspects of the problem in order to produce a mathematical model which will give a realistic solution

Task 2 – Hooke's law

- Students are asked to use given laws in physics to solve a traditional type of applied maths problem using differential equations



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Task 3 – A box to contain nuts and bolts

- This is the well known “max-box” problem applied to a ‘real’ world context.  Students have use the parameters of the box to find a solution to the ‘real’ world problem

Task 4 – The quality of beer

- William Gossett had a real problem relate to the Guinness brewery.  His model enabled a real problem to be solved.  The lecturer presented his solution to the students

Task 5 – The best red paint

- The lecturer used this ‘real’ context to construct a way of introducing statistical constructs to his students.  He explained  the solution to his students.

# Questions for discussion

- What is the nature of the ‘modelling’ in each of these tasks
- In what ways are the students supposed to be involved
- What can we say about the intentions for students’ learning?
- In what ways does the context relate to what the students are supposed to do? To what extent is it an essential part of the task?
- In what ways might the context motivate learning
- As a teachers, why would you use (or not use) a particular task?



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# Inquiry and Modelling

Inquiry-based tasks and modelling tasks both have the intention to engage students meaningfully in mathematics.

What are the key differences and where might we prefer to take a modelling approach?



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