Inspiring teaching by linking maths to industry

Chris Budd

IMI

UNIVERSITY OF BATH

Traditional ‘student view’ of maths?

- Maths is useless

•The only jobs for mathematicians are in accountancy and teaching

- Industry is full of people with oily rags

- Mathematicians have no concept of the real world

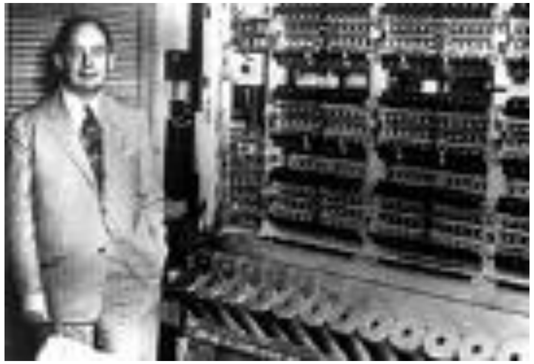
- Applied maths is either bad maths or not even maths at all

In contrast the reality is that good applications of maths change the world

Vectors, Maxwell, Radio, FFT, digital revolution, computers

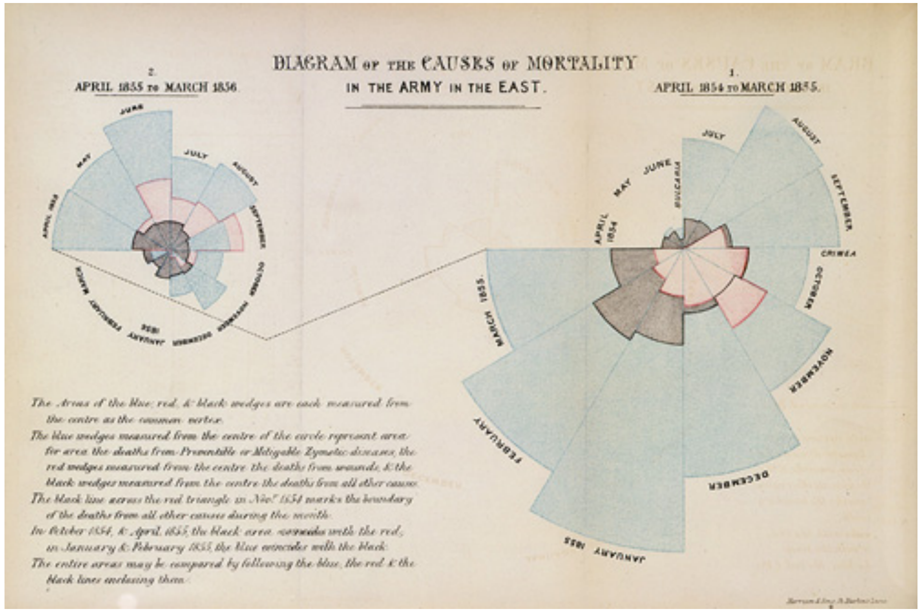
Google Matrices, SVD, page-rank

The computer



Mathematicians even save lives!

Florence Nightingale



Medical scanners, statistics, genetics, …

Indeed …

Much of industry has problems which can potentially be formulated, and solved using mathematics

Maths connects with all areas and knows no bounds or constraints!

Too few people recognize that the high technology so celebrated today is essentially a mathematical technology

Edward David, ex-president of Exxon R&D

Maths adds BILLIONS to the economy: UK Deloitte Report

And this can help our teaching at all levels by …

- Motivating students with real life examples
- Providing challenging problems for them to work on which develop the mathematics that they learn in their courses
- Encouraging them to learn new maths in the process
- Developing other skills such as team work,problem solving skills, presentation skills, research skills, interdisciplinary working, modelling, working with unstructured problems, rapid computing
- Giving examples of future careers which use maths

Which industries use math?

Traditional industrial users of math are

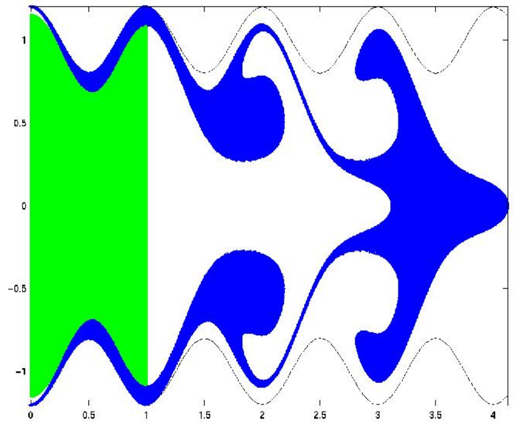
Telecommunications, aerospace, power generation, iron and steel, mining, oil, weather forecasting, security, defence, finance, forestry

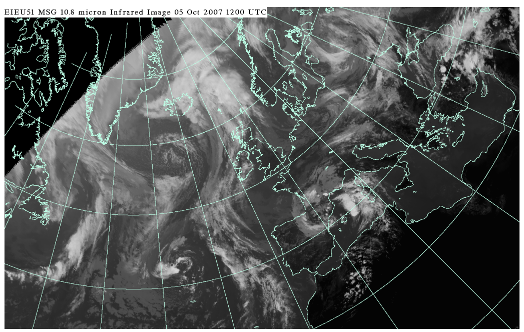
But they could equally well be …

Retail, food, zoos, sport, entertainment, media, forensic service, hospitals, air-sea-rescue, education, transport, risk, health, biomedical, environmental agencies, art, computer graphics, …

What sort of math does industry need from our students?

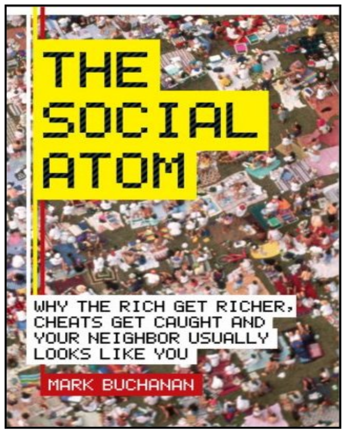
- Problem solving skills + logical skills
- Numeracy
- Data analysis
- Calculus
- Statistics and OR
- Differential equations
- Mechanics
- Matrices
- Complex numbers

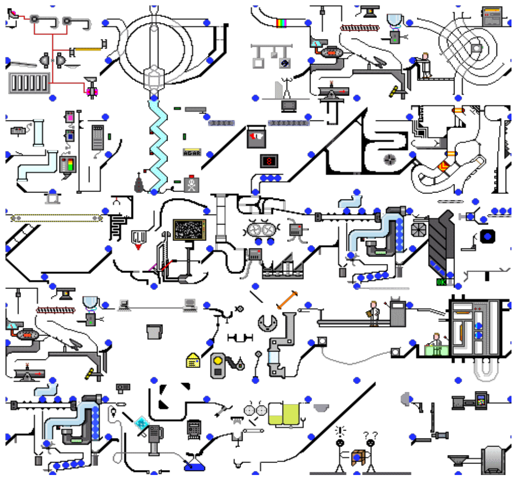




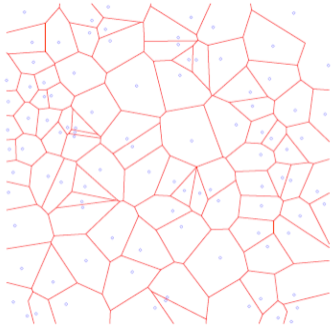
ALSO .. 21st century applications of maths will be driven by even more exotic industrial applications

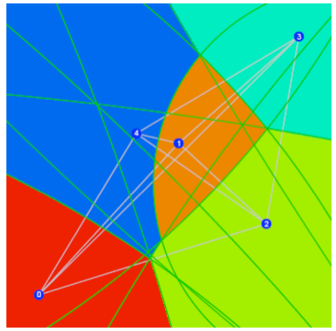
- Information/Bio-informatics/Genetics
- Commerce/retail sector
- Complexity
- People based activity
- BIG DATA eg. Facebook





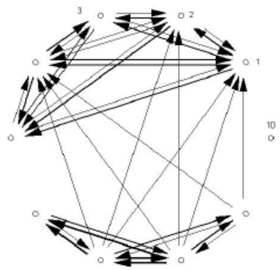
And this means that ever more maths will be needed in the future for industry





- Network theory
- Probability and uncertainty
- Computational maths
- Game theory

- Optimisation
- Number theory
- Algebraic topology



- In fact pretty much all of

‘pure math and statistics’

Introducing real world ‘industrial’ examples has many advantages in teaching

The challenges of industry make students think ‘out of the box’ and address new challenges

Leading to them discovering new maths in the process

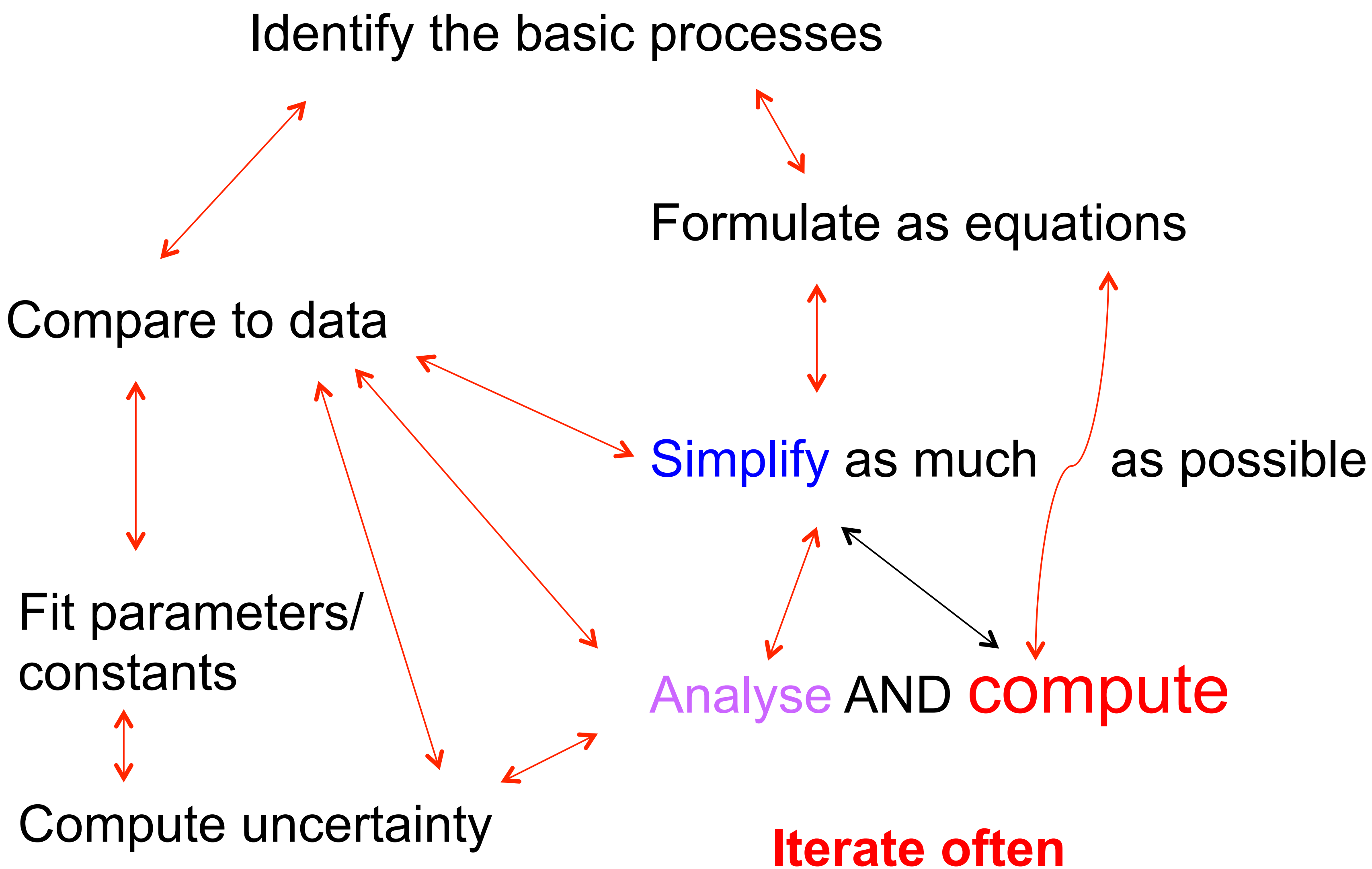
Which leads to great teaching examples and lots of student motivation both for students who will be serious users of math and those who will still use some in their careers

The modelling process (Theory)

This is a key aspect of using math to solve real world problems which we need to teach our students

- Identify the key issues, simplifying where necessary
- Formulate the system in mathematical terms
- Solve the system as well as possible, using computers if necessary
- Check against reality and correct where necessary
- Interpret your mathematical result in real world terms

My own experience .. A little hard to teach !



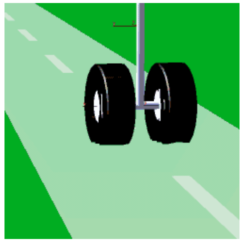
Iterate often

Once you have done this then …..

- Extend the model if needed .. Have you REALLY considered all of the issues?
- Do lots of what if experiments
- Discover things you never knew before (Neptune)
- Optimise your systems
- Change the world! (eg. Newton, Maxwell, Google, ..)
- BUT always take a good hard look at what you have done and

DON'T Eat the menu (often done!!)

Physical/Engineering/??biology problems



- Basic processes often quite well understood (ODEs/PDEs)

- Rescale/non-dimensionalise equations to identify small parameters to identify important processes

- Analyse these processes where possible:

perturbation methods, homogenisation, dimension reduction and exact solutions can be very helpful

- Never be afraid to compute as early as possible!

Social science problems (including finance and much of industry)

- Common and important
- MUCH less well understood
- Have to be creative in identifying the key processes involved

- Don’t aim for too much sophistication
- Get as much data as possible
- Always include stochastic effects
- COMPUTE

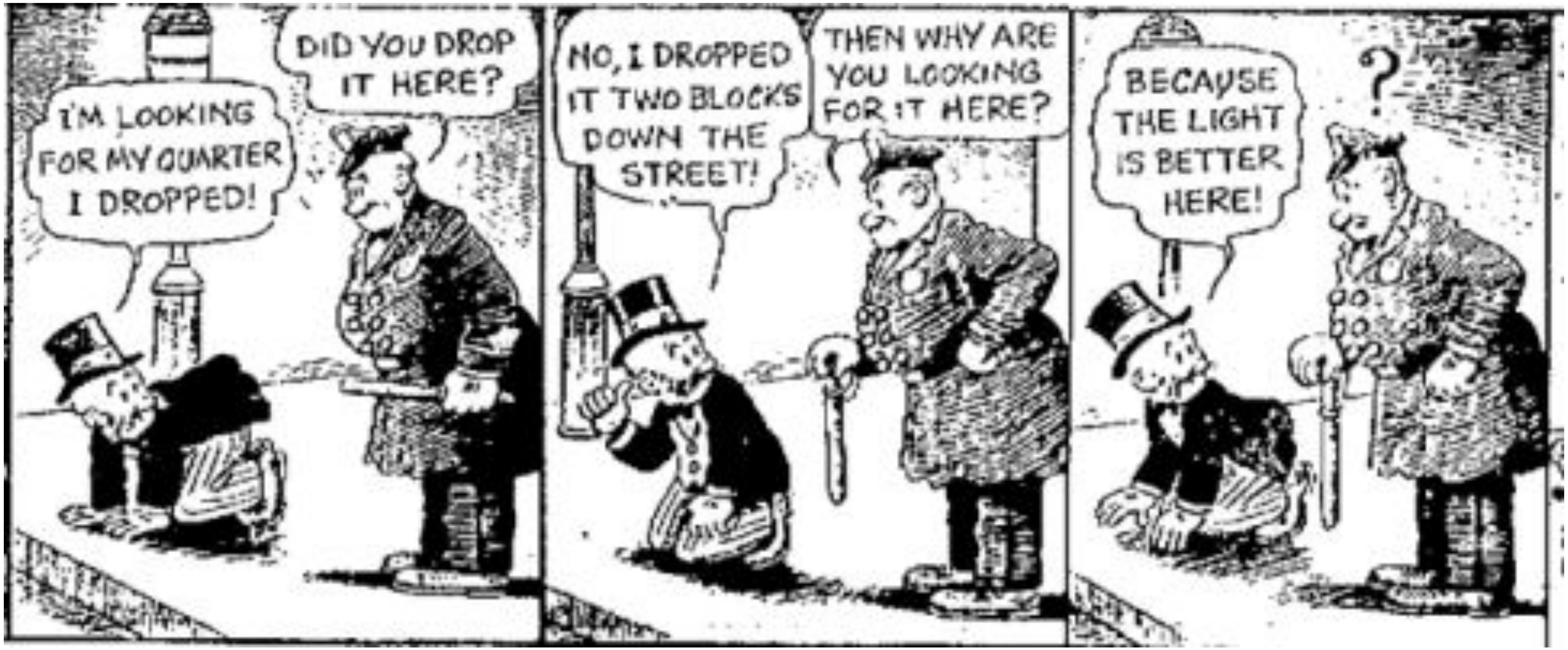
Big guiding principles for students

1. Make sure that you are solving the right problem (this may not be quite the problem originally asked)

2. Make sure that you solve the problem right

> **We thought that we had the answers, it was the questions we had wrong.** - Bono

But ….try not to be a mathematical drunkard



Examples of real world problems to challenge students with

Level one: Fermi Problems and Guesstimation

How many piano tuners in Oslo?
How much water in Norway's lakes?

Level two: Modelling traffic, Internet (Google), projectiles, planets, WiFi location, cancer treatment, bees, saving the whales, etc.

Level three: Modelling the climate, brain, evolution of the universe

So .. How can we including such examples into our courses

Three possible mechanisms

1. Whole courses based on Case Studies

   eg. MEI Critical Thinking (High school)

   Bath, Bristol, Greenwich, (Undergraduate)

   Oxford, Bath (Postgraduate dedicated courses)

2. Examples in relevant courses

   eg. Methods, mechanics, statistics, modelling, …

3. Study groups and camps eg. Dedicated weeks such as ESGI

1. Examples of whole courses

A. MEI Core and Critical-Maths

New course designed to provide motivational math for 16+ year olds who will not be doing a math based course at university

http://mei.org.uk/critical-maths

Course/resources designed to enable students to think about real life problems using mathematics.

Resources start by engaging the students in giving an initial opinion and then encourage them to think more deeply and to evaluate their initial thoughts.

Attempts to show how mathematics can be useful when making informed decisions

Example:

A fair comparison?

On board my Ryanair flight, the magazine tells me that Ryanair mishandles fewer bags than any other airline – 0.38 bags per 1000 passengers compared to the European average of 9.4 per 1000 passengers.

Is that an impressive difference

Discussion

Ryanair charge for checking in luggage – it cost £60 for my bag to travel in the hold.

Because of this, many Ryanair customers travel with cabin baggage only.

Is that enough to explain Ryanair's better record of bag mishandling?

Is bags per 1000 passengers a fair way of comparing

B: Bristol Undergraduate course:

Maths and Data Modelling

A 20 credit unit that runs in the first three years of the programme (out of 120 credits per year)

Focuses on open ended problems of an industrial nature

Very highly rated by students!

> *This was the most interesting unit in the entire year. I feel that I gained more wisdom from it than from all other units combined.*
>
> *A student*

Also very challenging because of open endedness…

> *Didn't really have any idea where to start on the project…*
>
> *Another student*

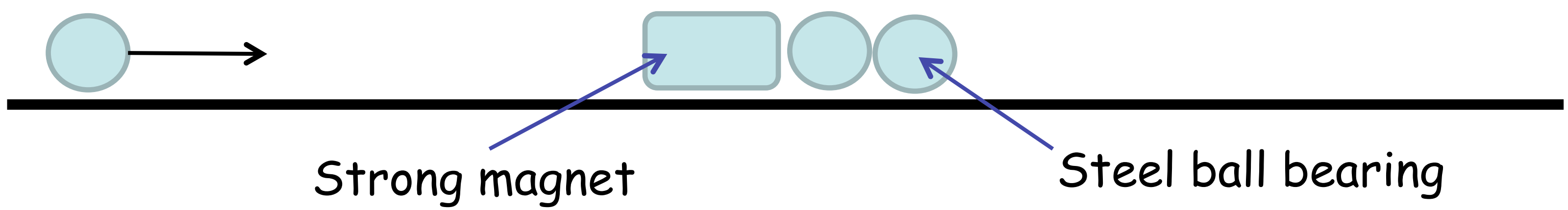
Bristol: Maths and Data Modelling

First year:

- Toy problems that can go a long way
    - How much toothpaste does a tube of toothpaste hold?
    - What is the best way to place spaces in a car park?
- Academics talking about what they do

Second year:

- More “real-world” problems



Bristol: Maths and Data Modelling

Third year:

- Anything goes…
- Use complete range of mathematical skills
- Independent thinking/problem solving

Great for motivating graduate-level mathematics

Good for making mistakes!

C: Oxford/Bath Doctoral Training Centres

Close contact between industry and students

Dedicated problem solving workshops/think tanks: ITTs

Projects directly with industry

Student internships

Industrially funded PhDs

2. Examples in courses

$$e\sigma T^4 = (1-a)S$$

Less ambitious but still very effective.

Can be used in calculus, mechanics, methods, statistics, …

Great to have a history of the example and to tell the story of how the maths really made an impact

A. Forensic mechanics Catching a speeding motorist (high school)

Was the car speeding?

Forensic evidence: collision damage,

witness statements,

skid marks

Evidence: s distance of skid

Cause: u speed

Other data: F brake force per unit mass

Mechanics links speed to distance

$$\boxed{s = \frac{u^2}{2F}}$$

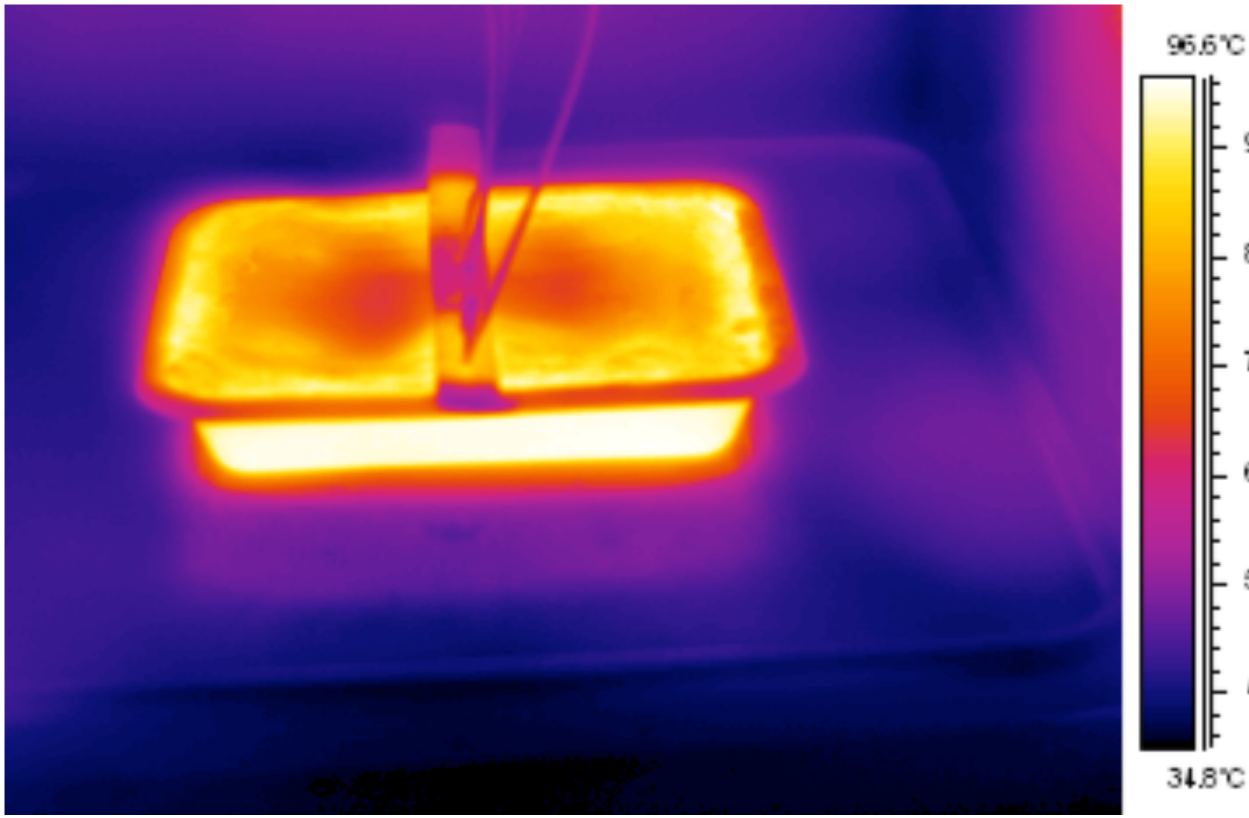
Given the distance math gives the speed

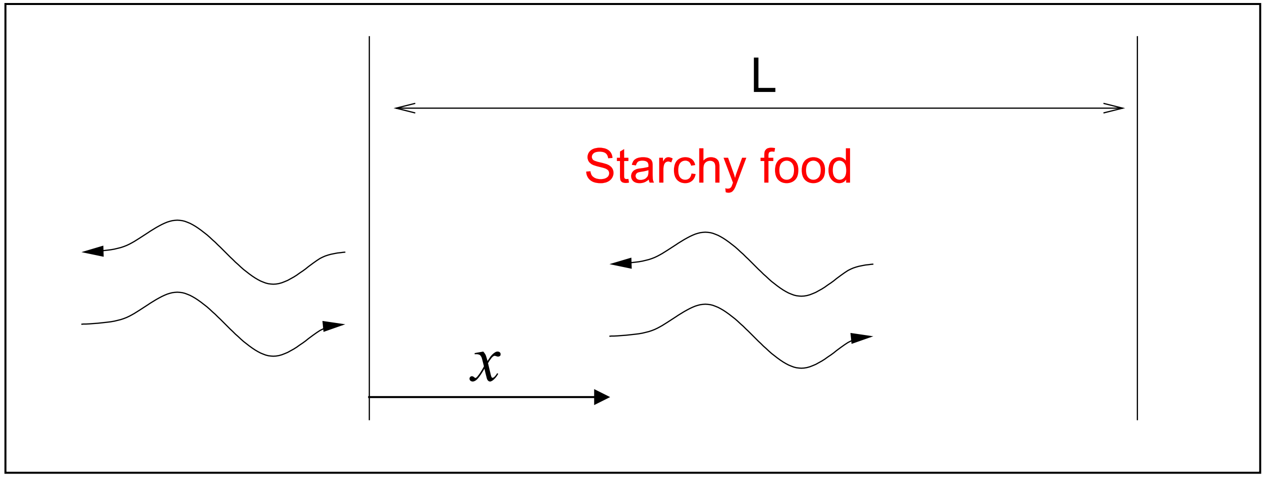
$$\boxed{u = \sqrt{2Fs}}$$

B: Microwave cooking (3rd year undergrad)

What gets hotter, the outside of the food or the inside?

Thermal image of surface of food after 5 minutes heating





L: Domain length: 2-14cm

d: Penetration depth: 8mm

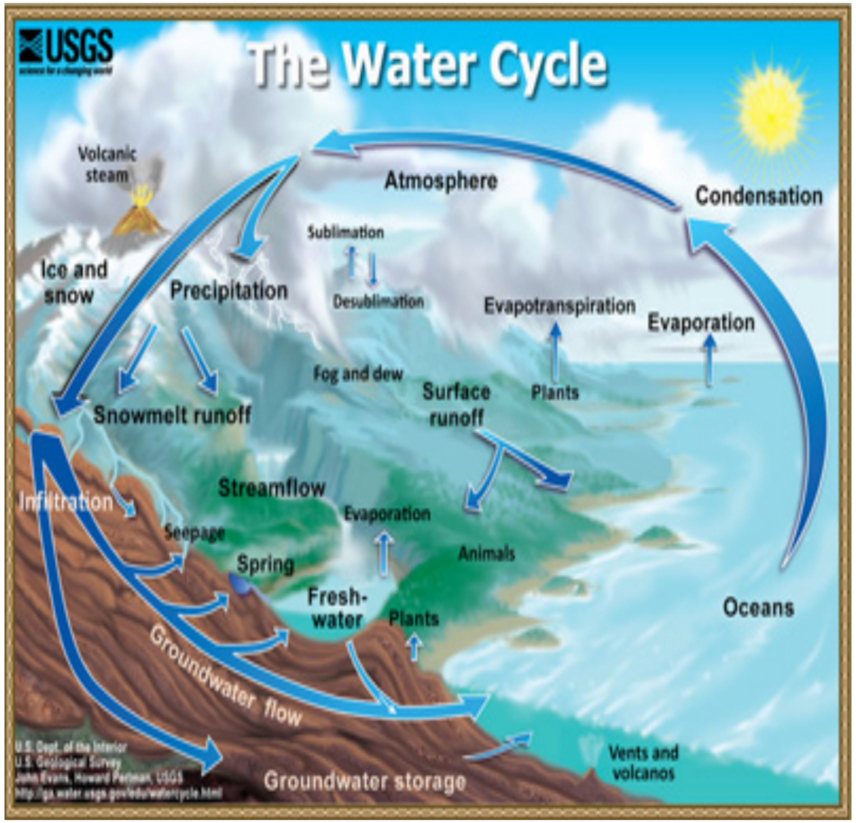
Solving Maxwell's equations for electric field predicts that the power absorbed decays exponentially. Temperature T satisfies a differential equation

$$\frac{\partial T}{\partial t} = k T_{xx} + P\, e^{-x/d}, \quad T(0,x) = T_0, \quad kT_x + h(T - T_0) = 0 \quad at \quad x = 0$$

Challenge .. Solve this .. (a) in general (b) steady state.

C: Climate (any level)

Simulation:



$$\frac{Du}{Dt} + 2f \times u + \frac{1}{\rho}\nabla p + g = \nu\nabla^2 u,$$

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho u) = 0,$$

$$C\frac{DT}{Dt} - \frac{RT}{\rho}\frac{D\rho}{Dt} = \kappa_h \nabla^2 T + S_h + LP,$$

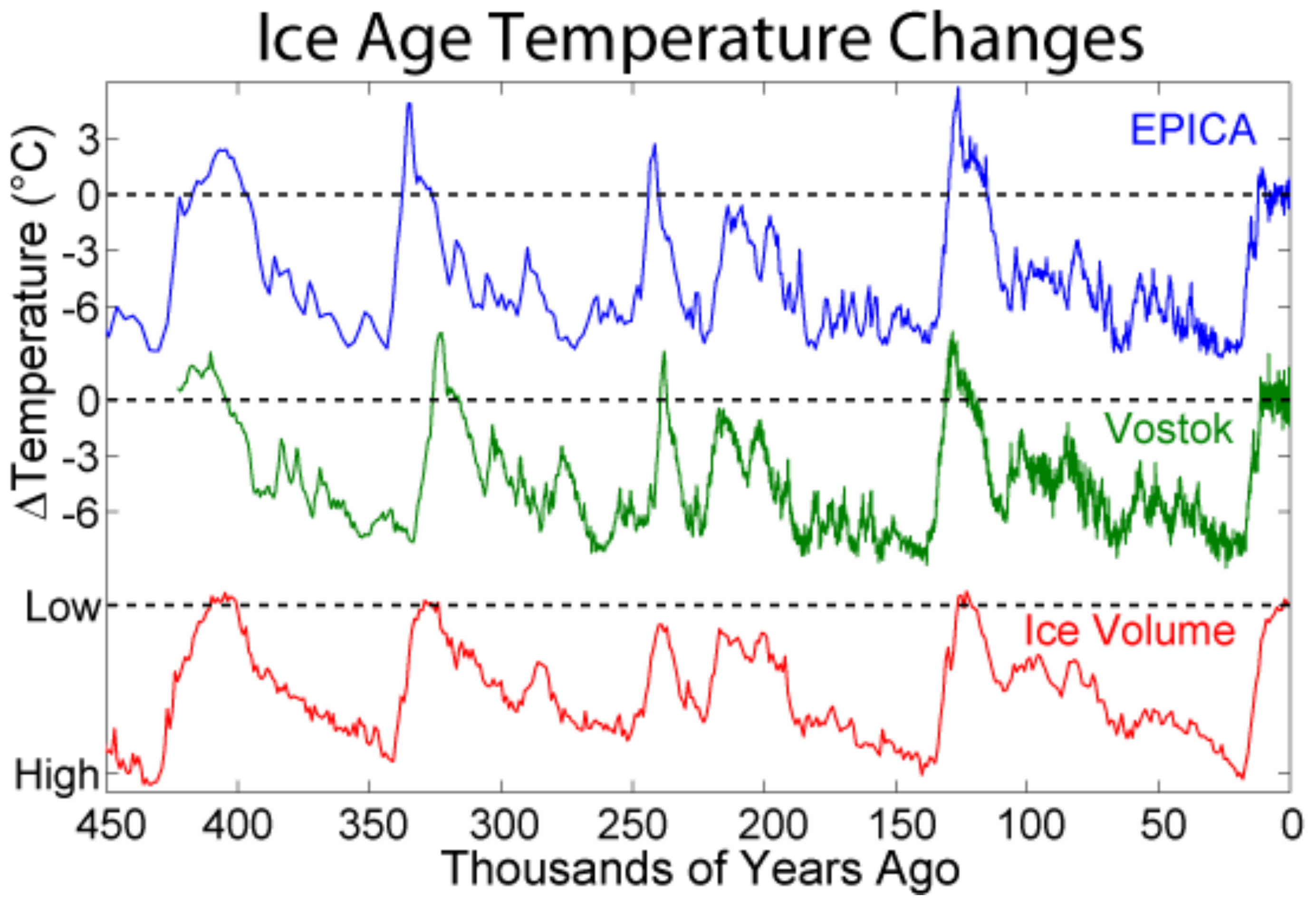
$$\frac{Dq}{Dt} = \kappa_q \nabla^2 q + S_q - P,$$

$$p = \rho RT.$$

For climate add in ice, CO2, ocean currents, vegetation, volcanos, solar variation, ….

Very complex. Hard to solve, hard to test, hard to interpret!

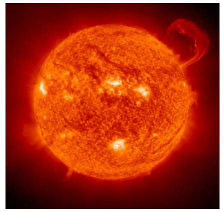
No real use in helping us to understand past climate



Looks regular .. Can a model predict this?

A simple model

Heat absorbed → $(1-a)S$



a Albedo: How well the earth reflects the Sun's rays

Heat radiated away → $e\sigma T^4$

e emissivity: How much energy is radiated into space

Balance these

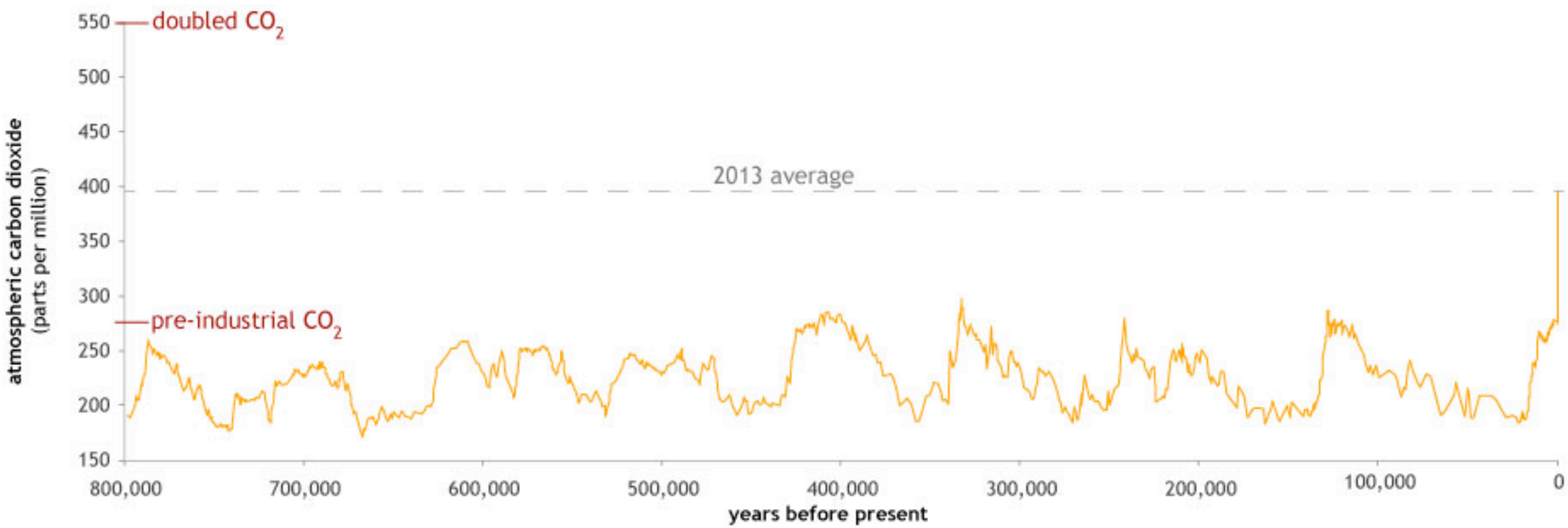
$$\boxed{e\sigma T^4 = (1-a)S}$$

Solve to find the temperature T

a depends upon Ice cover

e depends on Carbon Dioxide

Level of Carbon Dioxide (ppm)	Emissivity e_{CO2}
200	0.194
400	0.14
600	0.108
800	0.085



Model helps predict global warming but not much else

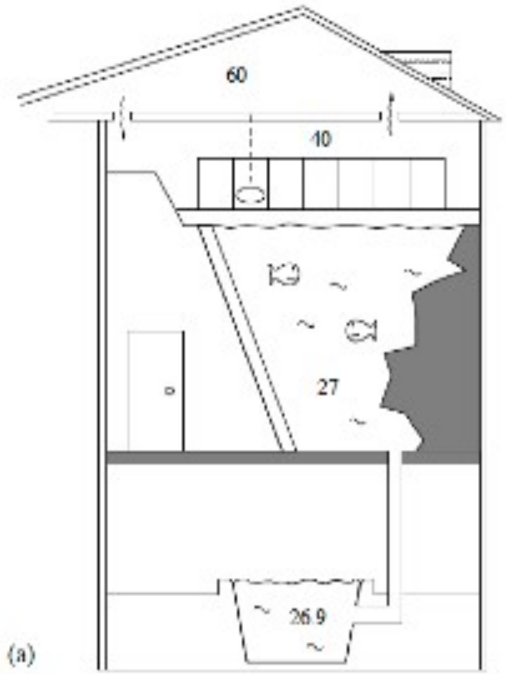
Need a hierarchy of models to do the job!

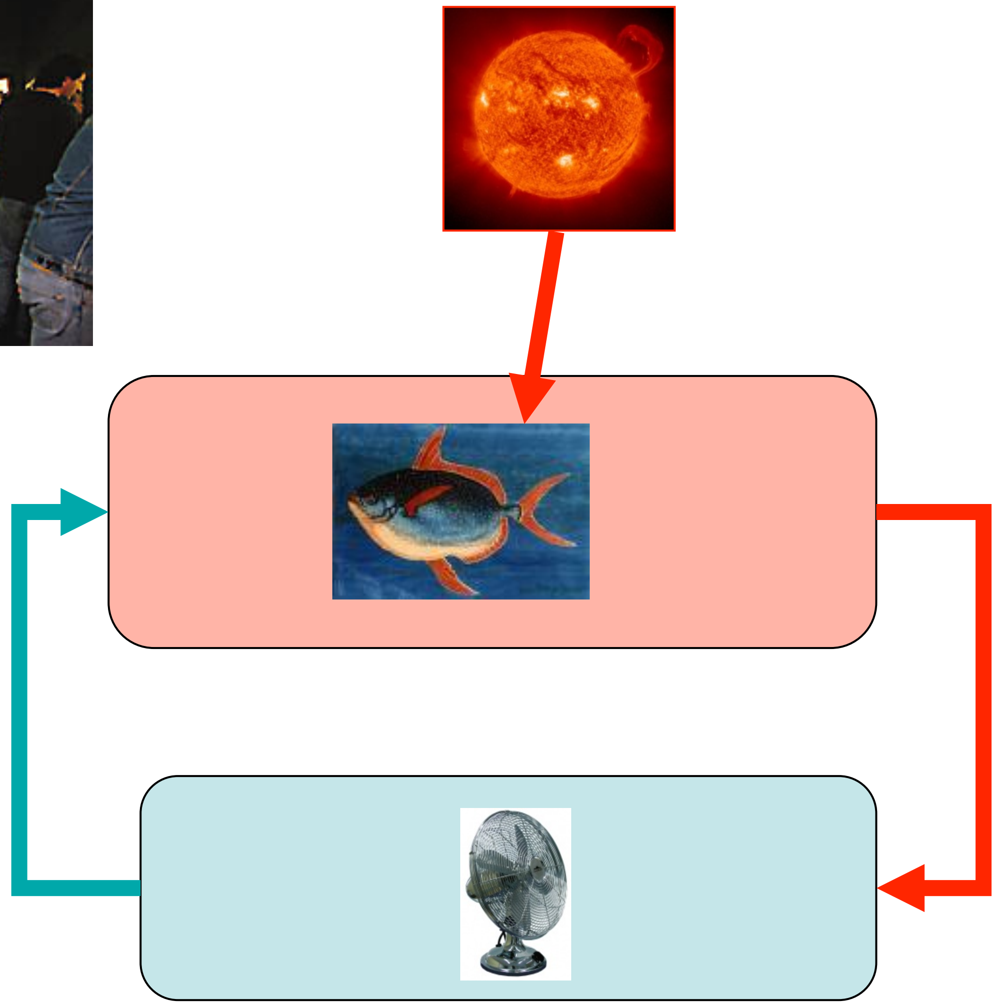
3. Training camps and study groups

Study Group Model (in use all over the world, including Norway) ESGI = European Study Group with Industry

- Bring academics, students and industrialists together
- Pose raw or processed industrial problems
- Work on the problems in teams
- Present results under time pressure
- Coupled with training camps IPSW,
  Modelling weeks, ITTs

ESGI Example: Temperature of Dutch Fish Tanks





Mathematics in Industry
Information Service

Past Study Groups

Note: Not all links refer to the original website. Some are copies of the original website and are hosted by either MIIS or the **Web Archive**.

Study Group reports. Not all reports are uploaded. Please check later if not available, or send copies to miis at maths.ox.ac.uk if you have them or can help in getting them. Thanks.

Clicking on the icon, when shown, leads to reports page on MIIS ePrint.

Name	Place	Date
European Study Group with Industy, 129	Denmark	Aug 21-25, 2017
Montreal Industrial Problem Solving Workshop, 8	Montreal, Canada	Aug 7-11, 2017
Third SGI - São Carlos, Brazil	São Carlos, Brazil	Jul 10-17, 2017
Modelling Camp at São Carlos, Brazil	São Carlos, Brazil	Jul 3-8, 2017
First Israeli Modelling Week	Nahariya, Israel	Jul 2-6 , 2017
Mathematics in Industry New Zealand 2017	New Zealand	Jun 26-30, 2017
33rd Annual Mathematical Problems in Industry (MPI) Workshop	New Jersey, US	Jun 19-23, 2017
European Study Group with Industry, 128	Limerick, Ireland	Jun 11-16,

Information
- Home
- About Study Groups
- News & media
- Sponsors
- Contact us

Study Groups
- Upcoming
- Past / Reports
- Discussion Forum

Other
- Mathematics in Medicine
- Networks

http://www.maths-in-industry.org/

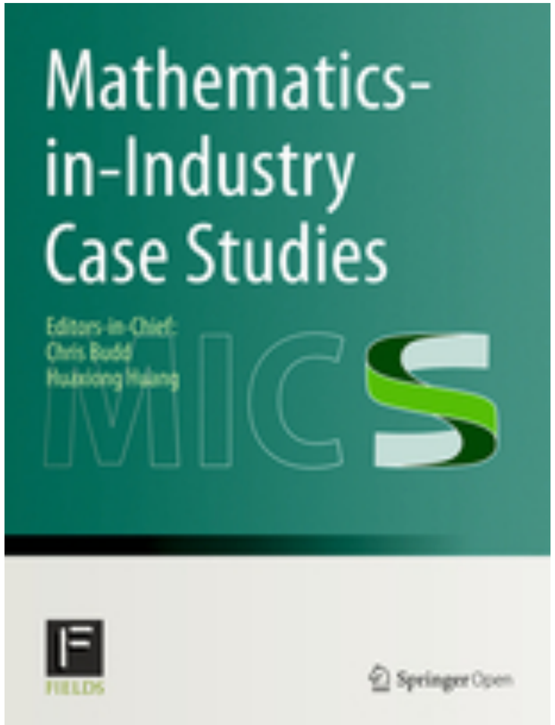
Facilitating and Training

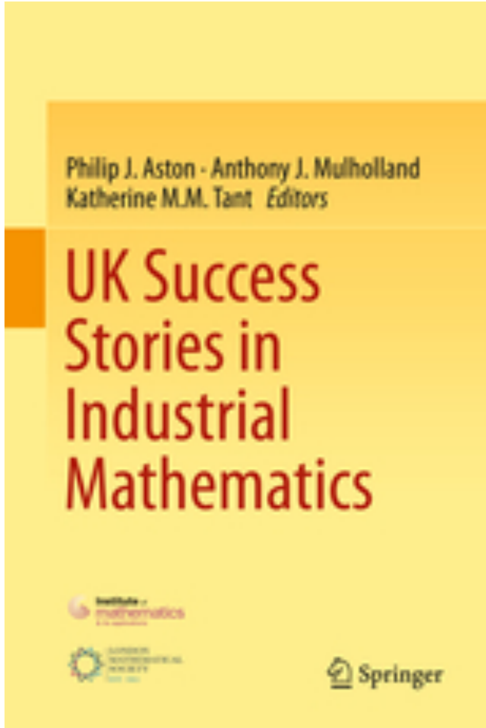
1. ECMI and MI-NET act to link projects between industry and universities

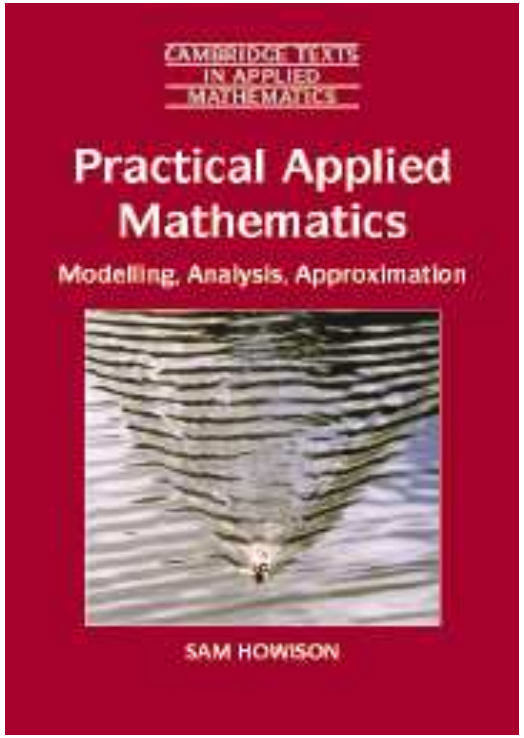
2. Many journals publish results of research originating in interesting study group problems and the Fields Institute has created the series Mathematics in Industry Case Studies (MICS) expressly for the publication of study group reports

3. ECMI runs Training sessions for PhD students prior to a study group workshop itself.

Some resources







http://www.maths-in-industry.org/miis/

http://www.mathscareers.org.uk/

Conclusion

- There is no substitute for using practical problems
- for both inspiring and motivating our students
- And also teaching them some fantastic maths in the process

The mathematical modelling process 2

What is a mathematical model?

Simulation: Detailed mathematical description of a problem with as many effects as possible. Often millions of lines of computer code (in C++ or Fortran) taking weeks to run

Gives numbers but not much insight

Model: Simplification containing the essential processes which can be analysed and computed on quickly (in Matlab/Python)

Gives insight and sometimes numbers

A model should be as simple as possible, and no simpler
A. Einstein