MatRIC Centre for Research, Innovation and Coordination of Mathematics Teaching

Centre of Excellence in Education

Supporting others to learn mathematics

A very brief guide for teaching assistants

Over recent decades we have witnessed amazing achievements in science, medicine and technology. For example, it is possible to land and operate robots on Mars, and there are great advances in discovering and developing treatments for diseases that have beset humankind for generations. Our mobile devices help us to stay connected and provide us with a library of information. It is astonishing what has been achieved through the scientific endeavour.

Curiosity rover at "Rocknest," in Gale Crater, Mars[i]

Despite these incredible advances in knowledge, exploration and engineering we have not solved the big problems in mathematics education:

POOR PERFORMANCE POOR PROGRESSION

FAILURE DROP-OUT

Research shows that more students fail at some point during their studies because of mathematics than for any other single reason.

For the last half-century, the world has witnessed the MATHEMATIZATION OF KNOWLEDGE. There are very few subject disciplines that do not make some use of mathematics in the pursuit of knowledge. Consequently, more students are required to study and learn mathematics than any other subject.

At the same time there is a DEMATHEMATIZATION OF SOCIETY as mathematics is 'hidden' within computers and calculators and thus perceived as irrelevant.

Research shows that many students are not able to apply the mathematical knowledge they have and put in into use when needed.

These notes are intended for those who will offer support to students who are seeking some help with their mathematics.

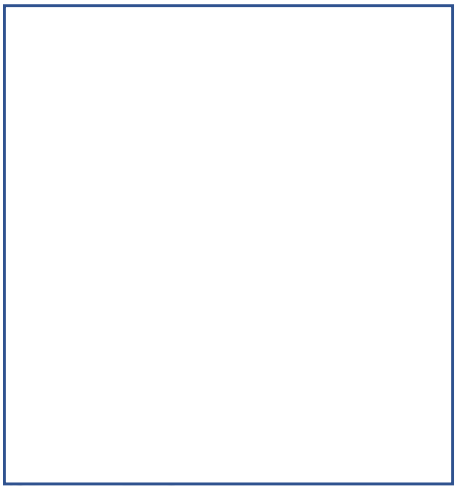
Mathematics Education Research is a relatively recent field of inquiry, international research journals in the field began to emerge about 50 years ago. Half a century of scholarship and inquiry make it possible to point to a number of fundamental principles and conditions related to success in teaching and learning mathematics …

Pre-existing knowledge is decisive: Mathematics is a 'hierarchical' subject in which ideas and concepts build on earlier ideas and concepts. New knowledge requires secure and sound existing knowledge, and new meanings needs to connect into existing meanings.

> *The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him[/her] accordingly.*
>
> *David P. Ausubel 1968.*

Motivation is essential: Learning mathematics is hard work, the learner needs to be motivated by having the prospect of success, through experiencing some reward – enjoyment, interest, usefulness – of the subject. Most students are required to learn mathematics as part of another subject (engineering, economics, etc.) they need to be able to see how the mathematics they must learn is relevant.

Informative, constructive feedback is crucial: Good feedback will inform the student about what s/he is doing right and will point to errors and suggest ways in which the student can explore and understand the reason for the error. Notice, it is feed**BACK** not feed**FORWARD,** the intention must be to lead the student to learn from failure, not to replace the failure with a readymade correct solution.



Mindset for success play an important role: A student who looks on a problem with the belief that s/he will learn and get stronger through the struggle is likely to engage. A student who looks at a problem and fears failure that will make her/him look 'dumb' is likely to avoid the challenge.

Deep thinking is much more effective in the long term than surface thinking: How is the student using her/his brain? Is s/he trying to memorise facts, which lead to short term success. Or is s/he trying to make the subject matter personally meaningful, to make sense and understand. In mathematics to appreciate the reasoning and connection with other related ideas results in knowledge that sticks and is ready for what comes later. It is important to connect with the 'big' ideas in mathematics such as pattern, variation, relation, function and transformation. Deep thinking takes time, there is no 'quick fix' for developing a strong understanding of concepts.

> *Knowledge is meaningful by definition. It is the meaningful product of a cognitive ("knowing") psychological process involving the interaction between "logically" (culturally) meaningful ideas, relevant background ("anchoring") ideas in particular learner's cognitive structure (or structure of his[/her] knowledge), and his[/her] mental "set" to learn meaningfully or to acquire and retain knowledge.*
>
> *David P. Ausubel 2000*

Students must be ready to be challenged: If it is easy there is nothing to learn. Students need to be prepared to get STUCK and to have a tool-box of strategies to help them get 'unstuck', such as make the problem simpler (try special cases, change the variables, add information, adapt the problem to an easier – more specialized form), find a similar example, explore the reason for being stuck. Do these before 'Googling', asking a friend, or asking an expert. Solving problems requires perseverance. The brain works like a muscle, the more it is exercised the stronger it becomes.

Students have expectations about teaching and learning: They may expect the teacher to make the subject 'easy', they may expect to solve problems within two or three minutes before giving up, they may expect the teacher to simply 'show' how a problem is solved and give the answer. Students have many years of experience of mathematics like this. But the good teacher does not make the mathematics 'easy', the good teacher makes it 'accessible'; real problems take time to solve – sometimes hours, or days, or even years for research mathematicians. The good teacher does not give the answer, the good teacher supports the student to work and solve the problem themselves, that way understanding and mathematical competence develops. The good teacher does not do the talking, but encourages the student to talk.

The power of assessment: Performance in assessments and examinations is essential for success and progression. Students want to pass their examinations and get the highest score possible. But examinations are not so good at assessing the nature of students' understanding or their competence at problem solving or modelling. So, project work, oral examinations and portfolios are used to give a better assessment of students' mathematical knowledge. Nevertheless, the traditional examination still has a very strong influence on what students will learn, and where emphasis is placed in teaching. Knowledge of skills and procedures assessed in examinations is important, but so are understanding, meaning, problem solving, modelling and learning strategies.

Teaching change is difficult and slow: After thirteen years in school, and some time at university, we learn not only mathematics, but from the example of many teachers, what teaching mathematics looks like. David Lortie refers to this as *'apprenticeship of observation'*. If we are going to improve on past performance we must teach better than our teachers, not just copy them. Trying out new ideas in teaching is not easy, early attempts often do not go well. Do not give up, think about what could be better and try again and again … and again – but always with a clear plan and goal – that students will understand and be independently successful learners.

Mathematics is difficult: The objects of mathematics are abstract concepts, mathematics is a study of ideas, which are human inventions mostly designed to solve real problems. Working at mathematics we do not work with the concepts directly, but with representations of the concepts. It is necessary to get beneath the representation to understand the concepts, it is helpful (perhaps essential) to work on concepts through several different representations (in words, symbols, diagrams, graphs and tables).

Do not worry about your difficulties in mathematics;
I can assure you that mine are still greater.

Albert Einstein

Things to keep in mind when teaching and supporting others to learn mathematics

When one reads or hears these suggestions, they may sound like self-evident truths. If that were the case, why does so much mathematics teaching appear to ignore them?

The one who does the mathematics is the one who learns mathematics

Mathematics is not a 'spectator sport'! To learn mathematics means to do mathematics. There needs to be variety of tasks, collaboration, discussion and application of mathematics in many different situations.

> *For the things we have to learn before we can do them, we learn by doing them.*
>
> *Aristotle.*

It does not help the student to do the mathematics for them. The teacher's task is to provide a 'scaffold' so that they can reach and do the mathematics for themselves.

Start from what the learner knows

Do not make assumptions about what a student knows already. The student must build on her/his own knowledge … not on the teacher's. This makes it difficult to know where to start with any suggestions. Try to get the student to explain what s/he understands, what s/he knows, what s/he thinks s/he should be doing and why s/he believes s/he is stuck. (*Helping by 'not helping'!*)

I cannot see through your eyes, you cannot see through mine

A student inspecting a mathematical statement is unlikely to interpret it in the same way that the teacher does. The teacher needs to work through the student's interpretation, that means trying to see the problem through the student's eyes. It is important that the student explains her/his interpretation to the teacher, and that the teacher questions the student's explanation to lead them to new or revised interpretation that will lead to a problem solution. It is also possible that in trying to explain the student begins to understand and can work her/his own way through the problem.

The learner must make her/his own meanings of mathematical objects

The teacher cannot learn for the student, the student must make her/his own meanings of the mathematics. The teacher can point to examples and to counter examples, but the student must explore and interpret the examples to make her/his own interpretation.

> *I cannot teach anybody anything, I can only make them think.*
>
> *Socrates*

Support the student to understand, do not reduce the mathematical challenge

Do not try to remove the problem from the student, support the student to deal with the problem. If the student is not challenged, there is nothing for her/him to learn. Strike a balance: too much challenge creates anxiety, too little challenge creates boredom.

Constructive feedback is the most effective way to support learning

Find where a student is going wrong and find out the reason. First ask the student to explain what s/he has done and understands. Get the student to think and talk about her/his interpretation, meaning or action. Note, a student who thinks differently from you may not be wrong, they may be creative, be prepared to be surprised! At the last resort give a brief explanation.

Breaking a task into simple steps does not lead to understanding

> *[I]t is a truism that there are very few single or simple adult acts that cannot be performed by a young child. In short, any more highly skilled activity can be decomposed into simpler components, each of which can be carried out by a less skilled operator. What higher skills require is that the component operations be combined.*
>
> *Jerome Bruner*

Get the student to look at the whole problem and suggest a strategy for solution – question the strategy. Get the student to work on those parts of the proposed solution strategy with which they feel confident. Provide support and possible direction where they lack confidence.

There can be several ways to solve a problem.

An effective strategy to support learning is to encourage students to find alternative ways to reach a solution.

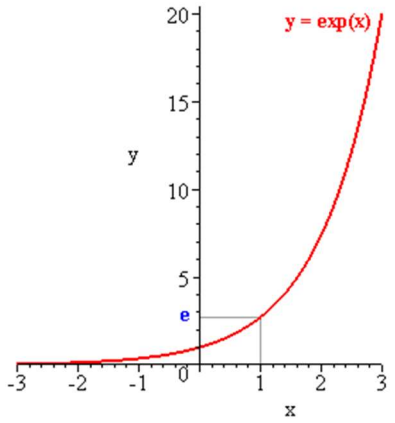
Mathematical concepts need to be experienced through different representations

Mathematics is populated by abstract ideas, we work on mathematics by manipulating the representations of the abstract ideas (an equation, a graph, a table of values, a collection of examples and counter examples). It is easy to confuse the abstract idea with the representation. A representation highlights parts of the abstract idea. It is necessary to engage with the abstract idea through several representations, and to be able to coordinate between the representations.

exp is a function whose rate of change equals the value of the function.

$$\frac{dy}{dx} = y$$

$$\mathbb{R} \to \mathbb{R}^+ : f : x \to e^x$$



The loudest voice one hears is one's own

If you want the student to hear – get her/him to do the talking. Do not expect students to follow long and complex explanations, better to get them to explain … may be a worked example in a textbook or their notes … certainly their own solutions to similar tasks whether correctly done or otherwise.

If you really want to learn something, teach it!

The chances are that by working as a teaching assistant your own knowledge and understanding of mathematics will grow and deepen. It is an experience shared by many mathematics teachers. If teaching is such a powerful aid to learning we must give our students, especially those that struggle, the opportunity to teach. Invite your students to explain to you and others what they [think they] have learned.

Problems in everyday and work settings are not like the problems met in mathematics textbooks. Real problems are fuzzy, messy, and ill-defined, *and meaningful*. Developing the competence to solve problems rather than just routine procedures lies at the heart of an education that prepares for LIFE!

Some strategies to use:

Try to connect with the problem the student experiences …

Ask the student to read the problem aloud to you … and listen**.**

Ask the student to explain what the problem or task is about.

Ask the student for a similar example in her/his notes – or the text book.

Ask the student to explain a worked example to you.

Find an easier task of the same structure and ask the student to solve it – working aloud. Have the student explain their solution. Then repeat with the more complex example to find out where the student gets stuck.

Consider using a different representation to solve a task: a graph in place of an equation, a diagram instead of text, a table of values in place of a symbolic expression of a function, etc.

Consider giving a contextual meaning for a task – situating it in an engineering or economics or other setting.

Good clear explanations and worked examples are essential, but they are only the beginning. **Understanding comes through working** at tasks and problems with increased complexity and challenge.

Develop an 'inquiry stance' – the important words are: Why? What? How? Where? When? and Who?

Practice is essential – it develops 'fluency' and helps to keep knowledge at the front of consciousness – which is where it needs to be for problem solving.

Be enthusiastic and optimistic. Enjoy mathematics, like problems!

Do not be afraid to say: 'I do not know', this is not a failure or sign of weakness.

Do not say, 'this is easy', it might be to you, it only leaves the student feeling inadequate and frustrated … and in any case, you are wrong 'it' is not easy. Falling off a log is easy, mathematics is difficult – that is why mathematics is more fun than falling off logs!

Do not work out 'oblig.' assignments for others.

[i] By NASA/JPL-Caltech/Malin Space Science SystemsDerivative work including grading, distortion correction, minor local adjustments, crop and rendering from tiff-file: Julian Herzog - http://photojournal.jpl.nasa.gov/catalog/PIA16239, Public Domain, https://commons.wikimedia.org/w/index.php?curid=22538554