Communicating Mathematical Meaning

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Introduction

The world is becoming more math-oriented,
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- The world is becoming more math-oriented, and there are opportunities for people who understand math:
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- The world is becoming more math-oriented, and there are opportunities for people who understand math:
  - Data scientist
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  - AI and machine learning
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Introduction

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Introduction

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Introduction

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Introduction

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Introduction

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Introduction

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Imagine future research directions:
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Introduction

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- Sports analytics
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Introduction

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  - Sports analytics
  - Movie animation
  - Technology consultant
  - Software engineer
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Imagine future research directions:

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Introduction

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  ▶ AI and machine learning
  ▶ Sports analytics
  ▶ Movie animation
  ▶ Technology consultant
  ▶ Software engineer
  ▶ Financial analyst
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Introduction

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- AI and machine learning
- Sports analytics
- Movie animation
- Technology consultant
- Software engineer
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Introduction

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  - Sports analytics
  - Movie animation
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Imagine future research directions:

- Communications: talking, stationary phones, internet, cell phones, ?
- Transportation:
Introduction

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  - Movie animation
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Imagine future research directions:

- Communications: talking, stationary phones, internet, cell phones, ?
- Transportation: walking,
Introduction

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- AI and machine learning
- Sports analytics
- Movie animation
- Technology consultant
- Software engineer
- Financial analyst
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Imagine future research directions:

- Communications: talking, stationary phones, internet, cell phones, ?
- Transportation: walking, cars,
Introduction

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Introduction

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- Transportation: walking, cars, planes, ?
- Societal problems:
Introduction

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Introduction

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Introduction

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Introduction

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Introduction

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Math and Ethics

Hello World: Being Human in the Age of Algorithms by Hannah Fry
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- **Mathematics for Social Justice: Resources for the College Classroom** edited by Gizem Karaali and Lily Khadjavi
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What the employers have said

They want to hire students with good mathematical skills, because of their
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▸ problem-solving skills
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- problem-solving skills
- ability to abstract
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► break complicated problems into solvable small pieces
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They want to hire students with good mathematical skills, because of their

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▶ ability to learn new things on their own
What the employers have said

They want to hire students with good mathematical skills, because of their

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▶ attention to detail
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They want to hire students with good mathematical skills, because of their

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- break complicated problems into solvable small pieces
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- attention to detail
- think of problems in a different way
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What the employers have said

They recommend that students should
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They recommend that students should

► learn to code
What the employers have said

They recommend that students should

- learn to code
- develop good communication skills
What the employers have said

They recommend that students should

- learn to code
- develop good communication skills
- do an undergraduate research project or a summer internship
What the employers have said

They recommend that students should

- learn to code
- develop good communication skills
- do an undergraduate research project or a summer internship
- learn about another discipline
Student error 1

A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side $x$ at each corner and then folding up the sides as in the figure. Express the volume $V$ of the box as a function of $x$ and give the domain of $V(x)$.

(a) $V(x) = 12 \cdot 20 \cdot x$ with domain $= (-\infty, \infty)$
(b) $V(x) = 12 \cdot 20 \cdot x^2$ with domain $= (-\infty, \infty)$
(c) $V(x) = x(20 - x)(12 - x)$ with domain $= (-\infty, \infty)$
(d) $V(x) = x(20 - x)(12 - x)$ with domain $= (0, 12)$
(e) $V(x) = x(20 - x)(12 - x)$ with domain $= (0, 20)$
(f) $V(x) = x(20 - 2x)(12 - 2x)$ with domain $= (-\infty, \infty)$
(g) $V(x) = x(20 - 2x)(12 - 2x)$ with domain $= (0, 6)$
(h) $V(x) = x(20 - 2x)(12 - 2x)$ with domain $= (0, 12)$
Student error 2

A cricket produces 142 chirps per minute at $50^\circ F$ and 152 chirps per minute at $100^\circ F$. Find a linear equation that models the temperature $T$ as a function of the number of chirps per minute $N$. That is, $T = mN + b$. What goes in the numerator of $m$? Is it the number of chirp values or temperature values?
A cricket produces 142 chirps per minute at $50^\circ F$ and 152 chirps per minute at $100^\circ F$. Find a linear equation that models the temperature $T$ as a function of the number of chirps per minute $N$.

That is, $T = \underline{\hphantom{0}} N + \underline{\hphantom{0}}$. The first blank represents a rational number $m$. What goes in the numerator of $m$? Is it the number of chirp values or temperature values?
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That is, $T = \underline{\hspace{2cm}} N + \underline{\hspace{2cm}}$. The first blank represents a rational number $m$. What goes in the numerator of $m$? Is it the number of chirp values or temperature values?
Certain ways of teaching can communicate incorrect math meaning to the students.
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(1) Rote learning without understanding:
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1) Rote learning without understanding: In the U.S pre-university students are taught to not to leave square roots in the denominator.

\[ \frac{3}{\sqrt{2}} \text{ is bad} \]
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- Students are not told why this is done.
- This communicates the message that an answer is wrong if you leave a square root in the denominator.
Certain ways of teaching can communicate incorrect math meaning to the student.

(2) Terminology:
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(2) Terminology: Improper fractions (without further explanation)
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(2) **Terminology:** Improper fractions (without further explanation)

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\frac{8}{3} \text{ is an improper fraction.}
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▶ The message students get is an improper fraction is not good.

▶ However, in the real world it is much easier to use improper fractions when working on problems.
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(3) Imprecise Language:
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A 1st grade elementary school teacher asks her students “Who can tell me how many of these numbers are divisible by 2?”

\[1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9\]
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None of the students respond, so the teacher calls on 7-year-old Fred Tate who replies
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(3) **Imprecise Language:** From the 1991 U.S. movie *Little Man Tate*

A 1st grade elementary school teacher asks her students “Who can tell me how many of these numbers are divisible by 2?”

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None of the students respond, so the teacher calls on 7-year-old Fred Tate who replies

“all of them!”

The teacher had been expecting the students to think “2, 4, 6, 8.”
What are other examples in which the way a concept is taught can lead to students’ misunderstanding?
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Group Discussion: What do you think?
Why do some students succeed and some fail university mathematics?
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Small Group Discussion: What do you think?
Why do some students succeed and some fail university mathematics?

Small Group Discussion: What do you think?

Large Group Discussion: What do you think?
Why do some students succeed and some fail university mathematics?

Small Group Discussion: What do you think?

Large Group Discussion: What do you think?

Some ideas:

▶ Most students have a stronger procedural knowledge of mathematics
Why do some students succeed and some fail university mathematics?

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Some ideas:
- Most students have a stronger procedural knowledge of mathematics
- Most students have a weaker conceptual understanding
Why do some students succeed and some fail university mathematics?

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Some ideas:

► Most students have a stronger procedural knowledge of mathematics
► Most students have a weaker conceptual understanding
► Most students feel comfortable using their strengths, but not their weaknesses in university mathematics courses. This hinders their deeper learning of mathematics.
Why do some students succeed and some fail university mathematics?

Small Group Discussion: What do you think?

Large Group Discussion: What do you think?

Some ideas:

► Most students have a stronger procedural knowledge of mathematics
► Most students have a weaker conceptual understanding
► Most students feel comfortable using their strengths, but not their weaknesses in university mathematics courses. This hinders their deeper learning of mathematics.
► Mathematically stronger students also begin with their strengths but then venture into less comfortable areas while studying.
Students’ understanding of limits

The concept of a limit is a fundamental mathematical notion.
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The word *limit* is used outside of mathematics. That can affect students’ understanding of the math notion of *limit*. 
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Group Discussion: How is the word “limit” used outside of mathematics?
Students’ understanding of limits

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**Group Discussion: How is the word “limit” used outside of mathematics?**

**Examples:**

- *Speed limit* - a legal rule not to be exceeded but people do exceed it.
Students’ understanding of limits

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**Group Discussion: How is the word “limit” used outside of mathematics?**

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- *Physical limit* - a boundary that is unlikely to be reached.
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Group Discussion: How is the word “limit” used outside of mathematics?

Examples:

- *Speed limit* - a legal rule not to be exceeded but people do exceed it.
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Group Discussion: How do these nonmath meanings of “limit” affect students?
What are the ways that mathematical “limit” is presented in a classroom?
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**Group Discussion: What do you think?**
What are the ways that mathematical “limit” is presented in a classroom?

Group Discussion: What do you think?

Three descriptions of limit:

- the informal definition involving notions of arbitrary closeness.
- the concept of values approaching a particular value.
- a function approach of outputs getting close to a value as inputs get close to a value.
- a graphical approach of finding limit values by tracing along a graph to see how outputs change get close to a value.
- the procedural approach including algorithms that produce a numerical value.
  - direct substitution
  - multiplying by the conjugate
  - L’Hospital’s Rule

Group Discussion: Are these descriptions connected or are they taught as separate items?
What are the ways that mathematical “limit” is presented in a classroom?

Group Discussion: What do you think?

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Researchers have shown that although students can “do” limit problems, they do not grasp the definition and when asked to explain what a limit is, they rely on explaining the procedures used to calculate the value.
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In addition to the procedural emphasis of their concept image of limit, students often show several common misconceptions of limits while taking introductory calculus.
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**Group Discussion:** What are ways that students mistakenly think of limits?
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**Example:** Which is larger $0.9$ or $1$?

In addition to the procedural emphasis of their concept image of limit, students often show several common misconceptions of limits while taking introductory calculus.

**Group Discussion:** What are ways that students mistakenly think of limits?

Students may develop misconceptions of the limit:

- limit as a bound,
- limit as an approximation of the function value, and
- limit value as equal to the function value.
Teaching to better communicate mathematical meaning

Group Discussion: What are different ways to teach a math concept?
Teaching to better communicate mathematical meaning

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Group Discussion: What are advantages and disadvantages of each?
Teaching to better communicate mathematical meaning

Group Discussion: What are different ways to teach a math concept?

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Group Discussion: How can we teach to better communicate mathematical meaning?
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Researchers have suggested:

1. Provide opportunities for students to be reflective learners.
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3. Help students develop their own examples to make sense of concepts.
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Researchers have suggested:

1. Provide opportunities for students to be reflective learners.
2. Integrate various representations in the curriculum emphasizing the strengths and weakness of each in the context of the problem being solved.
3. Help students develop their own examples to make sense of concepts.
4. Discuss similarities and difference among concepts such as limit, continuity and differentiability.
Thank you!
Thank you!

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