A study on different teaching approaches to some Linear Algebra concepts using tutorial videos as a research tool

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Abstract

The present research project is a continuation of a previous MatRIC project, in which I studied the production of short videos in mathematics and the didactic quality of their content. In the present study I used feedback from a reference panel on two sets of designed and recorded mathematical videos in linear algebra, to gain knowledge about how the mathematical content in teaching videos may be classified and which content and approach is more suitable for engineering students. Results provide information about differences between the two sets of videos, but divergences about how these differences can be characterized. As part of the research project, a pilot study where the videos were tested on two groups of students was carried out, and some reactions on the different approaches in the videos are reported. Results indicate that even if many students prefer one of the two sets of videos, the other set may lead to deeper knowledge of linear algebra concepts. This, however, calls for a more thorough data collection in order to investigate the effects closer.

Introduction

The research project reported here investigates two different approaches to a linear algebra concept, taught in a series of videos. The theme is *bases of general vector spaces*, which is perceived by students as difficult. As stated by Dorier and Sierpinska, “students tend to think in practical rather than theoretical ways” (2001, p. 209), which may be difficult with vector spaces beyond R, R^2 and R^3. The main target group for the videos are students that need to take a course in linear algebra as part of their studies to become engineers. I sought to gain knowledge about how the mathematical content can be classified and which content is suitable for engineering students. To capture the teaching, I used a sequence of videos so different participants (colleagues, students) could watch and evaluate these. In a previous MatRIC project, two sets of videos were designed and recorded, the sets presenting the same mathematical topics but with slightly different approaches. These sets of videos were distributed to a reference group, consisting of professionals in undergraduate mathematics and mathematics education, to gain feedback about their different approaches. As part of this investigation, a question about what is emphasized as conceptual and procedural approaches in a teaching video came up. To contribute to an answer to this question, additional collected data were analyzed and discussed, results of which were presented at the NORMA 17 conference in Stockholm (Rensaa & Vos, 2017).

To continue the research project, I draw on previous analyses and feedback about content of teaching videos (Rensaa & Vos, 2017) in addition to data collected in order to gain better insight into how different approaches to mathematical content are apprehended by users. Data were collected from a reference group where members have been asked to give feedback on the two sets of videos. Additionally, the videos have been tested in a class of engineering students. The implementation with students has been arranged as a pilot study, to log reactions to the slightly different ways of presenting the mathematical content.

Method

Videos

The design and making of videos was done with a comparative aim, striving to satisfy guidelines for multimedia (Mayer, 2005): Plan for the use of space, show the speaker's face, make rather short videos (< 7 min.). Two parallel sets of videos were made, called Group 1 and Group 2, each containing nine recordings. Four of the videos were common in both sets, for instance the introduction and the summing up videos. Five videos were diverging between the sets, engaging with the content in different ways. Details about the design is given in a previous MatRIC research report (http://www.matric.no/articles/52).

How feedback from a reference panel was collected

Feedback from a workshop at a MatRIC conference

The data collection from a workshop at a MatRIC conference is described in detail in the previous MatRIC report (http://www.matric.no/articles/52), while analysis of data, discussion and results were given in the NORMA 17 proceedings (Rensaa & Vos, 2017).

The selection of a reference panel

The selection of a reference panel to gain feedback about the two sets of videos was done with two aims. One was to have both members who produce mathematical videos themselves, and members who did not. The other was to have both members being professors in mathematics and in mathematics education. These groups are usually not disjoint, but that was unproblematic. The most important thing was to have some variation of background of the people in the reference panel, which was obtained. All together eight professors took part in this investigation.

The design of a questionnaire

The most feasible way of collecting data from the reference panel was to design a questionnaire with questions for the panel to answer by email. The questions referred to each of the videos in Group 1 and Group 2, presenting the mathematical content in slightly different ways, asking for reflections about these. Some questions were about the mathematical content, some about what each member of the reference panel found to be best suited for engineering students and some were combinations of these. Additionally, some questions about the two sets of videos in Group 1 and Group 2 as a whole were asked. The questionnaire was sent to each of the members in the reference panel – together with the videos in Group 1 and Group 2 – and the members responded by submitting written answers.

How to collect feedback from students in class

The students in an engineering education - for which I was the teacher in their linear algebra course - was split randomly in two groups, with about 30 students in each group. These groups were spread between 3 classrooms that contained computers if the students wanted to use these to watch the videos. The first group of students got a list of videos to watch where videos from the Group 1-set of videos were introduced. The students were not asked to judge the video content but to respond to given tasks in order to capture what type of knowledge they had gained from watching the videos. However, in order not to treat the students in class differently by giving them unequal teaching offers, the students were subsequently introduced to the Group 2-sets of videos. Thus, a typical line of work for a student in this first group would be

1. To watch a video from the Group 1-set of videos
2. To solve a task connected to the content of the video
3. To explain some theme introduced in the video
4. To watch the parallel video from the Group 2-set of videos, introducing the same theme slightly different
5. Finally to answer if the previous explanation of the theme had changed after having watched the Group 2-video.

The second group of students were given a similar line of work to be done, but with videos to watch in opposite order. Thus, these students watched a video from Group 2 first, then solved a task and explained a topic before watching the parallel video from Group 1.

Since the investigation on students was a pilot study where I mainly wanted to test their reaction to the scheme, I decided that submission of solutions to the tasks given was to be voluntary. I did, however, log events during the implementation in class as I was helping around during the complete video-watching and task-solving session.

Results

Selection of two videos to illustrate results

The feedback from the reference panel and from the students were rich but somewhat diverging. This is not surprising as the number of videos to give feedback on was quite big. The videos dealt with different aspects enlightening ‘bases of general vector spaces’, embracing topics like linear independence, bases with representations and address systems. Thus, I have selected one theme – dealt with in two parallel videos - to report on responses by the reference panel and students. The task dealt with in both these videos was the following:

The standard basis for P_3 is $\{1, x, x^2, x^3\}$.

Determine if

$$p_1(x) = 1, p_2(x) = x, p_3(x) = 3x^2 - 1, p_4(x) = 5x^3 - 3x$$

form an alternative basis for P_3.

This task was solved in slightly different ways in the two parallel videos. In Video 1 I utilized the definition of basis directly by testing for linear independency and spanning of the set of polynomials.

In Video 2 I chose an arbitrary polynomial, $p(x) = 5 - 4x + 2x^2 + 7x^3$, and showed that all terms in this polynomial could be described by our p_1 - p_4 polynomials given in the task;

$$1 = 1p_1$$

$$-4x = -4p_2, \text{ etc}$$

Initial interpretation in conceptual and procedural terms

The line of work in Video 2, where the given polynomials $p_1 - p_4$ were used to express an arbitrary polynomial, may be interpreted as visualizing what being a basis means. In accordance with definitions provided by researchers in mathematics education originating from Hiebert (1986), this may be read as teaching for conceptual knowledge, shortened as "using a conceptual approach". I draw on a working definition of conceptual and procedural knowledge about engineering students' learning provided by Bergsten and colleagues, saying that:

"A conceptual approach includes translations between verbal, visual (graphical), numerical and formal/algebraic mathematical expressions (representations); linking relationships; and interpretations and applications of concepts (for example, by way of diagrams) to mathematical situations.

A procedural approach includes (symbolic and numerical) calculations, employing (given) rules, algorithms, formulae and symbols" (Bergsten, Engelbrecht, & Kågesten, 2015, pp. 981-982).

Video 1, in which steps in the definition of a basis were utilized, may be regarded more in line with employing algorithms to solve the problem.

Feedback from the reference panel

The reference panel interpreted the two videos slightly different. As for Video 1, utilizing the definition, one in the panel commented on this as 'the standard one' – which may be in line with a routine way of solving the task. However, most of the others stated this video to be clearer, the best and the one being easiest to understand. The other video, Video 2, got some critique; like "tricky manipulations of coefficients", but was also stated to be illuminating. Thus, I got some diverging interpretations of the videos and could not make definite conclusions about them.

Feedback from students

In class

The video project was carried out in class according to plans. The students were still somewhat 'shy' as this was early in the course. Thus, there were not too many discussions in class. Nevertheless, this was how it was; the video project had to come this early since 'general vector spaces' are used throughout the course. Against this backdrop, however, there was a noticeable difference between the groups. Students in Group 2, who were to express an arbitrary polynomial in P_2 by p_1, p_2, p_3, struggled more than the other group. Much more tutoring work was required in the Group 2 classroom. One student in Group 2 expressed it like this, after having seen Video 2:

"Why are you doing it in this way? I don't want to do it like that [choose a polynomial and try to express it by the candidate for a basis]. I feel that this is much more bothersome than following the requirements of the definition of a basis like the textbook does. It becomes way too difficult to follow your approach. Besides, I did not understand why you initially chose one particular polynomial that you wanted to express".

I explained that the particular polynomial was just an example and that the argument could be generalized to a general polynomial – done at the end of the video. Still, the student decided to solve the task 'the textbook way', drawing on the definition including linear independence and spanning, saying something about preferring to follow sequential steps.

After class

Two weeks after the teaching video project was implemented, a student came to my office. He asked me to explain the progress in Video 2. He said that since he had been in Group 1, he had solved the task given in class after having watched Video 1, but was then satisfied and had not seen Video 2 during the implementation in class. Now, however, he had seen Video 2 and could not understand what was actually going on. He stated a problem with the concept of basis:

RJ: "What does it mean to have a basis of a vector space?

S: "That one, with as few vectors as possible, describes the vector space; as many as one needs, no more"

RJ: "*Yes, that is a good explanation. How come that you find it difficult to give such an answer after having seen Video 2?"*

S: …

Since the student did not answer, I decided that I just could ask him directly about the two videos:

RJ: "Which of the videos, Video 1 or Video 2, did you find to be the best?"

S: "The first one"

RJ: "*Why?"*

S: "The second was difficult to understand. Why did you just take a polynomial?"

RJ: "I tried to make things simpler. I could have used a general polynomial,

$p(x) = a + bx + cx^2 + dx^3$ *[I grabbed a pencil and wrote the general polynomial on a piece of paper], would that have been better?*

S: *"No, that would have been more confusing…*
I think I have realized that I have problems understanding what P_2 really is…"

Problems with understanding P_2 is not unusual, general vector spaces are found difficult by students (Dorier & Sierpinska, 2001), but the present student had not reflected on his understanding of P_2 while watching Video 1 and solving the subsequent task earlier.

Conclusion

My research interest is to gain knowledge about how the mathematical content in teaching videos are classified and which content is more suitable for engineering students.
A common agreement about the set of videos cannot be deduced from the results in the present project. Feedback from professionals at a conference, feedback from the reference panel and feedback from the students diverge in interpretations. As for the two sets of videos discussed here, the reference panel interpreted them as different, but their reading of how this difference occurred was not the same.
Not surprisingly, the engineering students signaled a preference to draw on Video 1 that followed the steps in the definition of a basis to show that p_1 - p_4 constitute an alternative basis to P_3. However, for one of the students the approach in Video 2 encouraged reflections about what a basis actually is. Students may not realise themselves which approach is most engaging to them.

In order to gain more knowledge about the different approaches in the videos, a more thorough data collection needs to be done in a next phase. Discussions during implementation in class need to be filmed to capture more of the students' reflexions. In addition, obligatory submissions of answers to tasks need to be required. In the pilot study, nearly all replies to the tasks came from Group 1 where Video 1 was watched first. Thus, a nuanced picture was missing.

References

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