Differential Equations in Mathematical Biology

- Anatomy of some applications -

Jorge Duarte

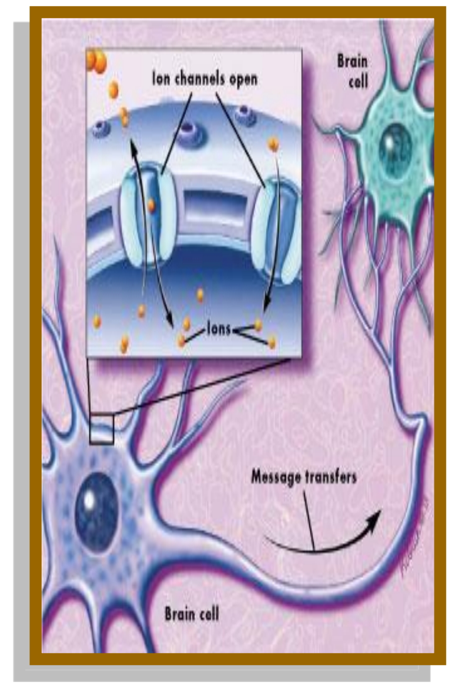
Engineering Superior Institute of Lisbon - Portugal

MatRIC

Modelling

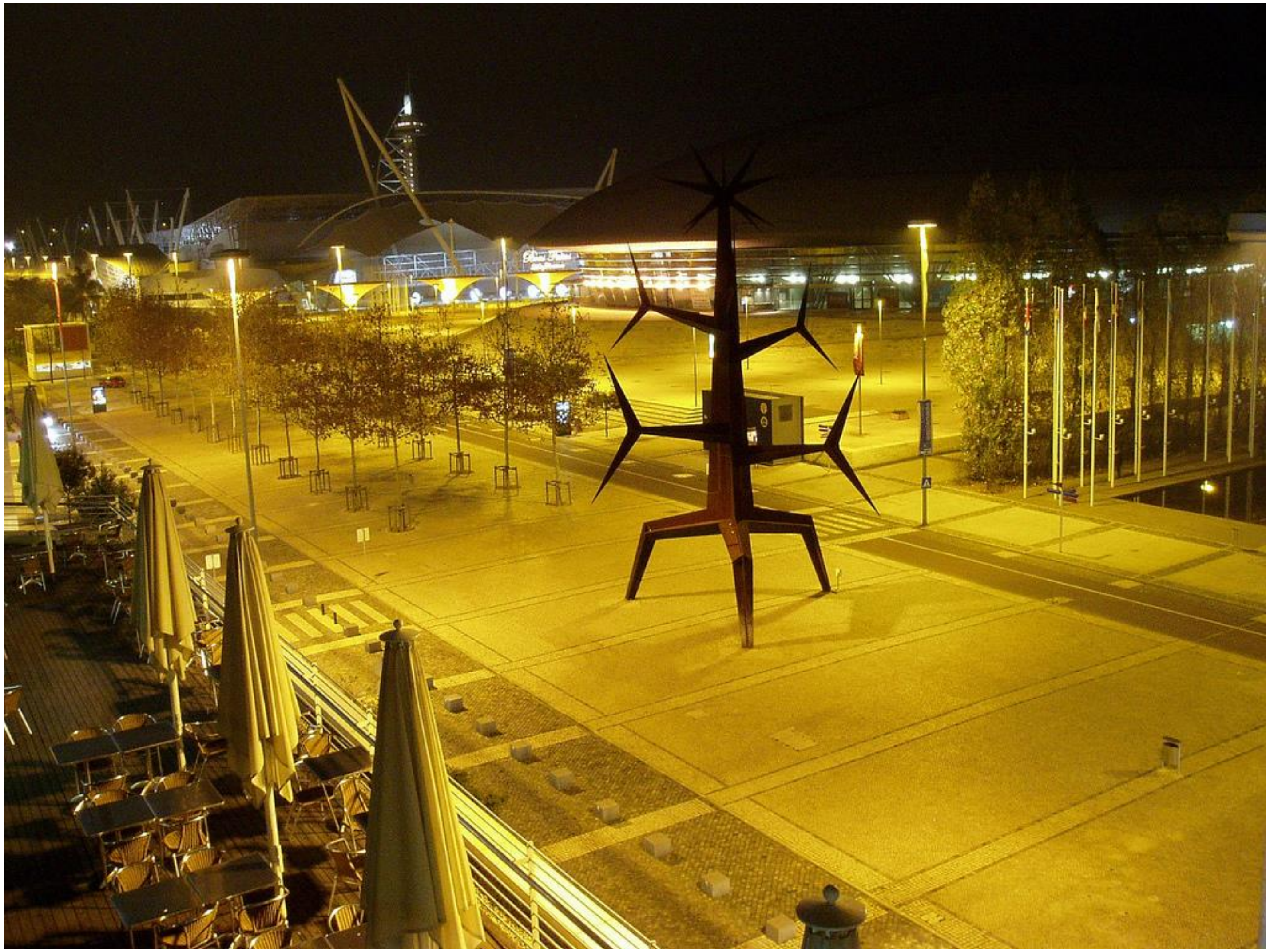
Colloquium

2016



Lisbon by day...

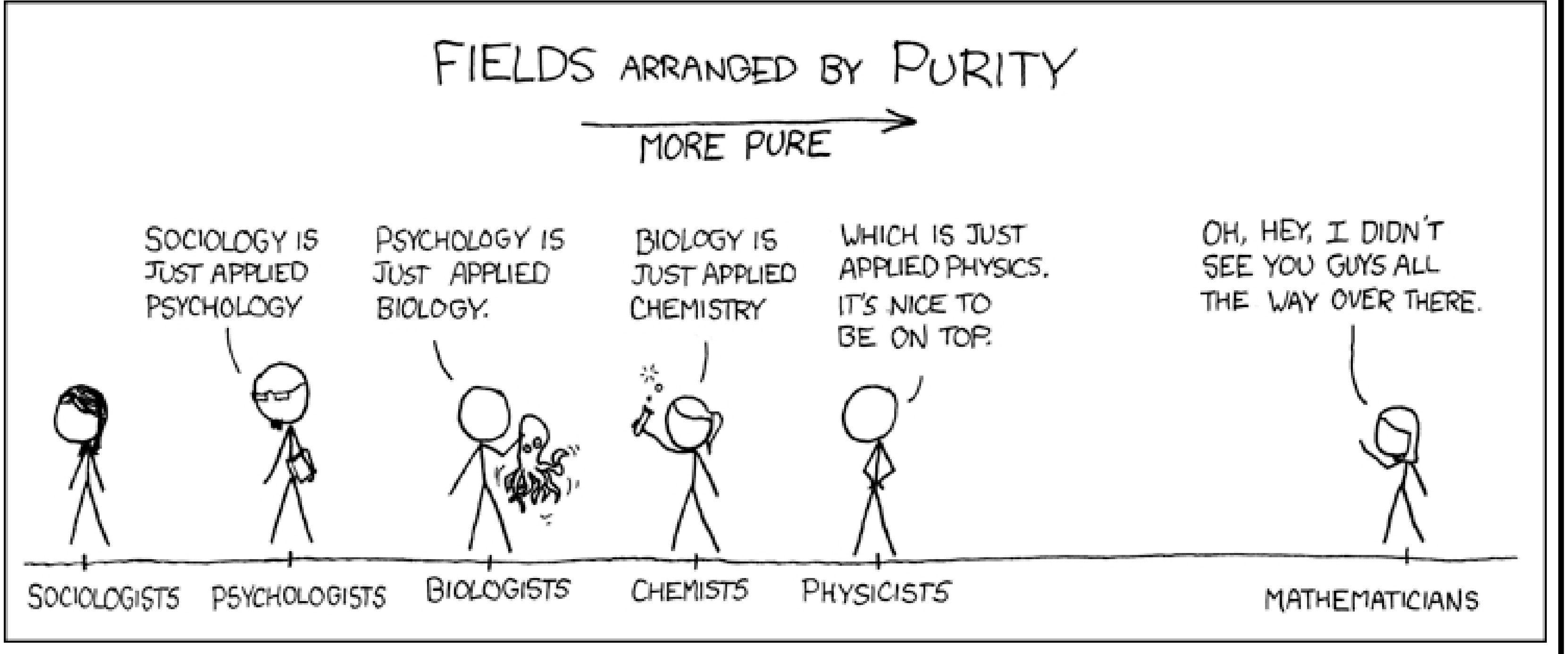
and by night…



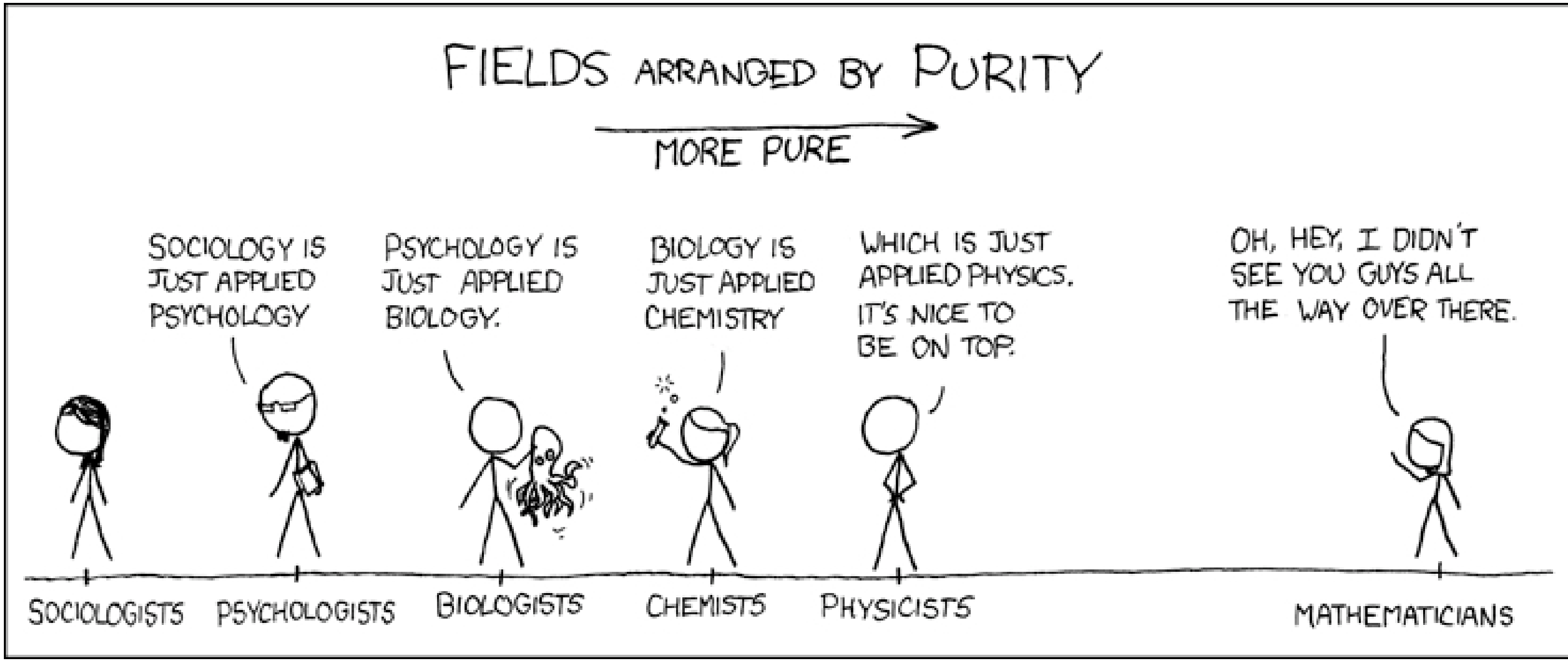
Aims for this talk

- Mathematics / other sciences
- Differential equations and biological rhythms
- Wonders of the nonlinear world
- Mathematical concepts in use
- Teaching / research - a multidisciplinary approach
- Key encouraging aspects / Maths matters

Mathematics / other sciences



Mathematics / other sciences



Question: Is the interdisciplinary work really difficult?

Our body in numbers

- We have around **42 billion blood vessels**.
  (about 160,000 km: 4 times around the Earth's equator, half way to the moon).

- The **heart pumps 8,000 liters of blood each day** (800 buckets).
  (During a lifetime around 88 Olympic swimming pools).

- Most of us will cry **68 liters of tears** (7 buckets).
  Certainly there will be gender differences :)

- We are constantly replacing our bones.
  The equivalent of **12 skeletons of new bone during a lifetime**.

- We take **500 million breaths** and inhale around
  300 million liters of air during our lifetime.

And many more...

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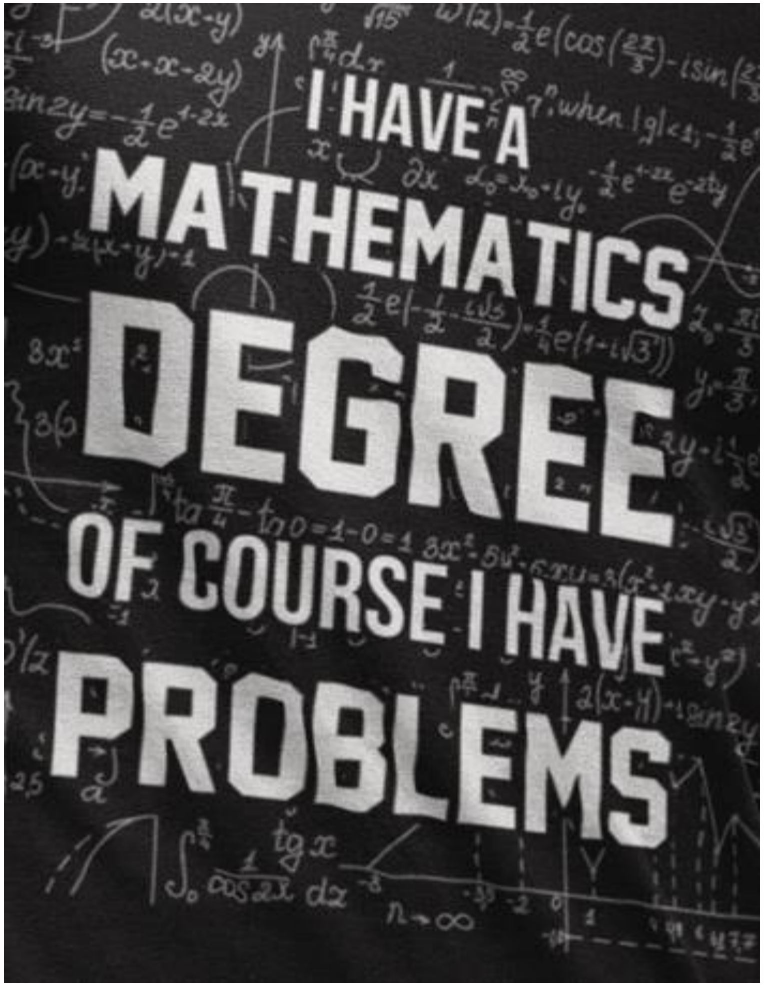
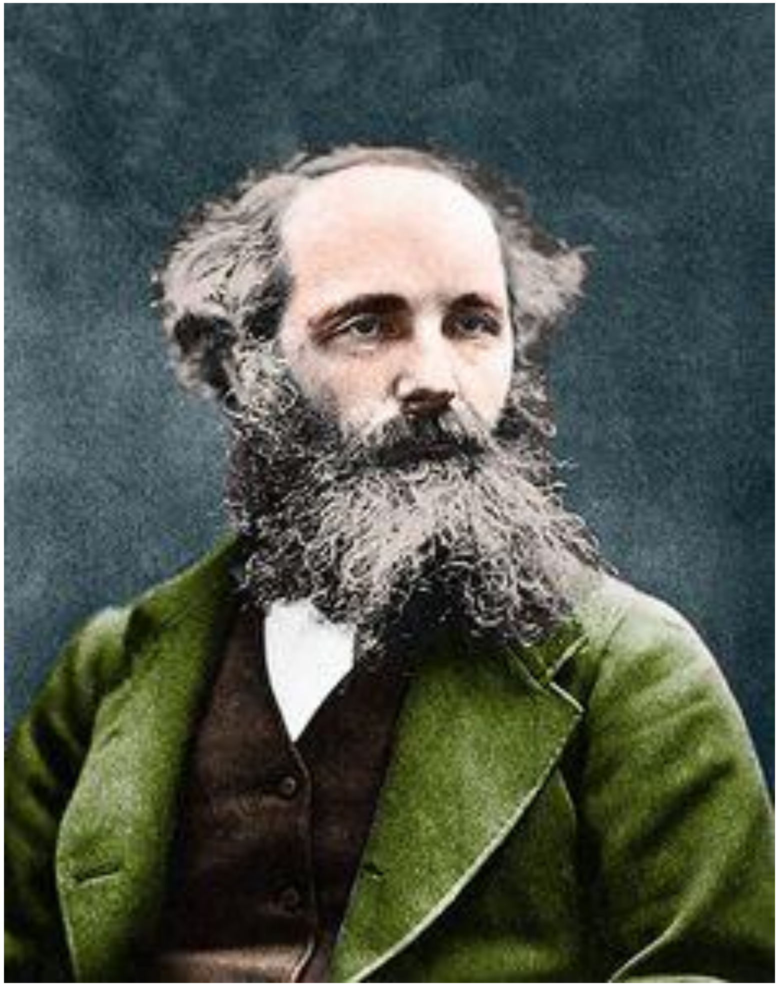
Quite impressive, isn't it?

Do you have problems 





“There is nothing more practical than a good theory.”

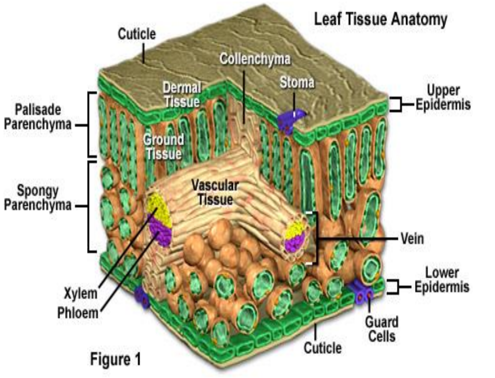
(James Clerk Maxwell (1831-79))

Differential equations and biological rhythms

The cellular growth

“Cells are matter that dance.”

(Uri Alon, *An Introduction to Systems Biology)*



- Suppose that the cell maintains a cubic shape.
- Starting from a certain volume v_0 at instant $t = 0$, the cell absorve continuously nutrients through the exterior membrane, which determines the increasing of the volume. This volume is $v(t)$, at each instant t $(t \geq 0)$.
- We identify the cellular growth per unit of time with the area of the exterior membrane of the cell (that is, the total area of the six faces of the cube).

- We express the total area of the cubic surface as a function of the volume

Let l be the length of the cube's side, so the volume v is

$$v = l^3 \quad \Rightarrow \quad l = v^{\frac{1}{3}}.$$

Since the total area of the six faces of the cube is given by

$a = 6\,l^2$ we have $a = 6\,v^{\frac{2}{3}}$.

- Thus, for $t \geq 0$ and $\Delta t > 0$ we will have

$$v(t+\Delta t) \approx v(t) + 6\,[v(t)]^{\frac{2}{3}}\ \Delta t$$

$$v(t+\Delta t) - v(t) \approx 6\,[v(t)]^{\frac{2}{3}}\ \Delta t$$

$$\frac{v(t+\Delta t)-v(t)}{\Delta t} \approx 6\,[v(t)]^{\frac{2}{3}}$$

Taking the limite when $\Delta t \to 0$,

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$$\frac{v(t+\Delta t) - v(t)}{\Delta t} \approx 6\,[v(t)]^{\frac{2}{3}}$$

Taking the limite when $\Delta t \to 0$,

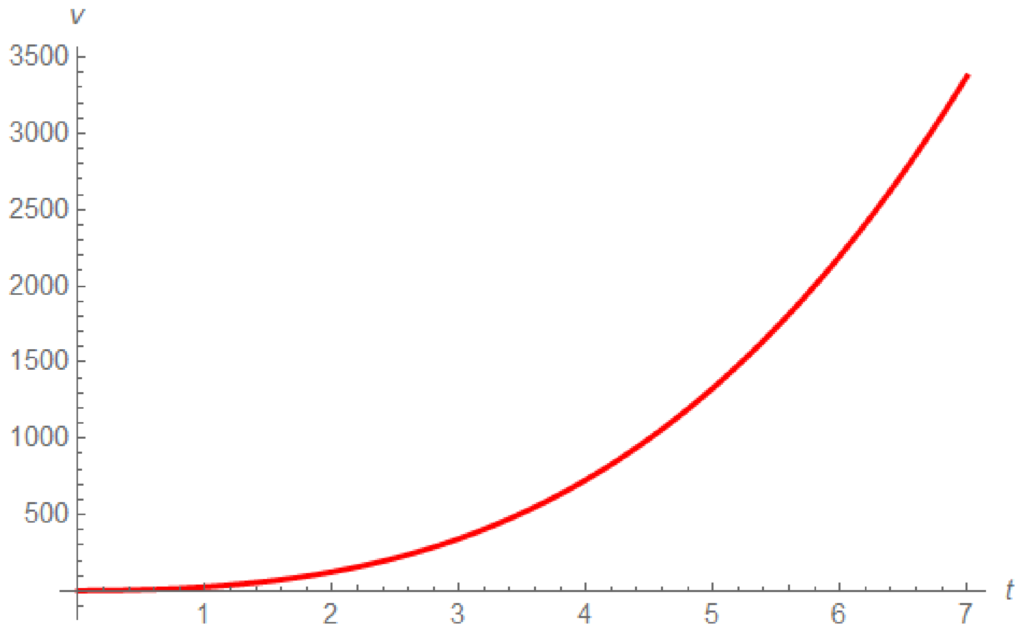
we obtain

$$v' = 6\,v^{\frac{2}{3}}$$

- A particular solution of this differential equation is

$$v(t) = (2t+1)^3$$

- The graph of this function is



A story of a detective

- There are problems, solved using differential equations, where information seems to be missing. The following situation is just an example of that...

....

"A body of a victim was found at 23:30 in a office which temperature we suppose constant and equal to 8,2º C.

At the moment the body was found, its temperature was 32,8º C, and after one hour was 31,6º C.

Considering that the normal temperature of the body is 36,8º C and that it is valid the Newton's law of cooling, i. e., the variation of the temperature of a body is proportional to the difference between the temperature of the body and the environment,

what time did the crime occur?

....

- Let $x(t)$ be the temperature of the body at time t and t_0 the instant when the crime was commited. So, we have

$$x(t_0) = 36{,}8^{\text{o}}\, C.$$

- Being $t_1 = 23{,}50$ the instant when the body was found, we know that

$$x(t_1) = 32{,}8^{\text{o}}\, C \quad \text{and} \quad x(t_2) = 31{,}6^{\text{o}}\, C, \quad \text{where } t_2 = t_1 + 1 = 24{,}5.$$

- Now, let us consider $t \geq 0$ and $\Delta t \geq 0$.

- Since the variation of the temperature per unit of time is equal to k times the difference between the temperature of the body and the temperature of the environment,

  $(x(t) - 8,2)$ at instant t,

  in Δt units of time, the variation of the temperature will be multiplied by Δt and we write

  $$x(t + \Delta t) \approxeq x(t) - k\,(x(t) - 8,2)\,\Delta t, \quad \text{for some} \quad k > 0,$$

  that is,

  $$\frac{x(t+\Delta t) - x(t)}{\Delta t} \approxeq -k\,(x(t) - 8,2).$$

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  that is,

  $$\frac{x(t+\Delta t) - x(t)}{\Delta t} \approxeq -k\ (x(t) - 8,2).$$

- With $\Delta t \to 0$, we obtain the differencial equation (model of our problem)

$$x'(t) = -k\ (x(t) - 8,2), \quad \text{for some} \quad k > 0.$$

- Taking in consideration the given initial conditions, the solution of the differential equation is

$$x(t) = 24,6\, e^{-0,05(t-23,5)} + 8,2.$$

- Since the temperature of the body, at the moment of the crime, was $x(t_0) = 36,8^{o}\, C$,

$$36,8 = 24,6\, e^{-0.05(t-23,5)} + 8,2$$

$$28,6 = 24,6\, e^{-0,05(t-23,5)}$$

$$t_0 \cong 20,4868.$$

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$$28,6 = 24,6\, e^{-0,05(t-23,5)}$$

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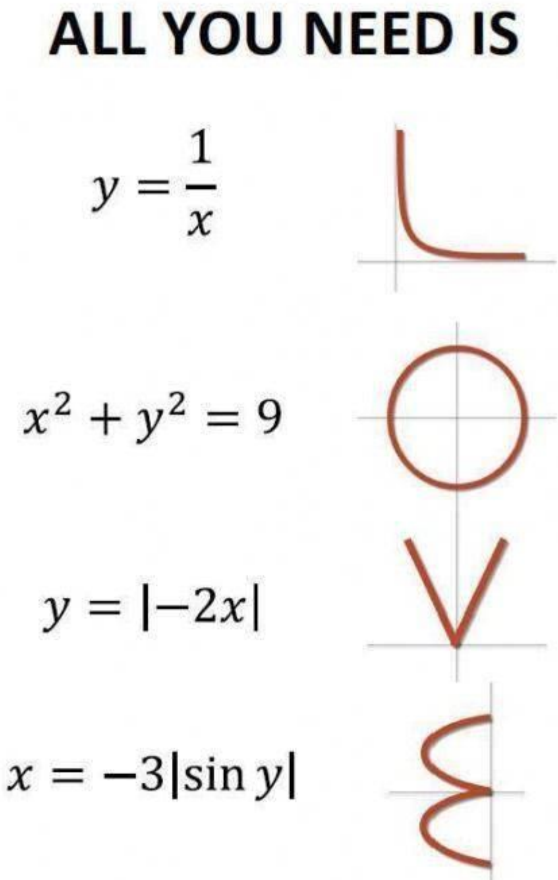
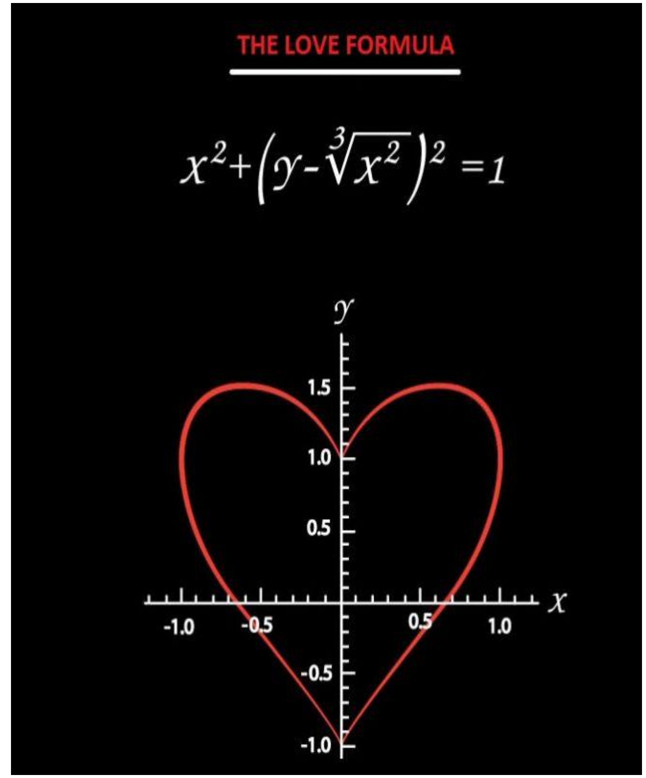
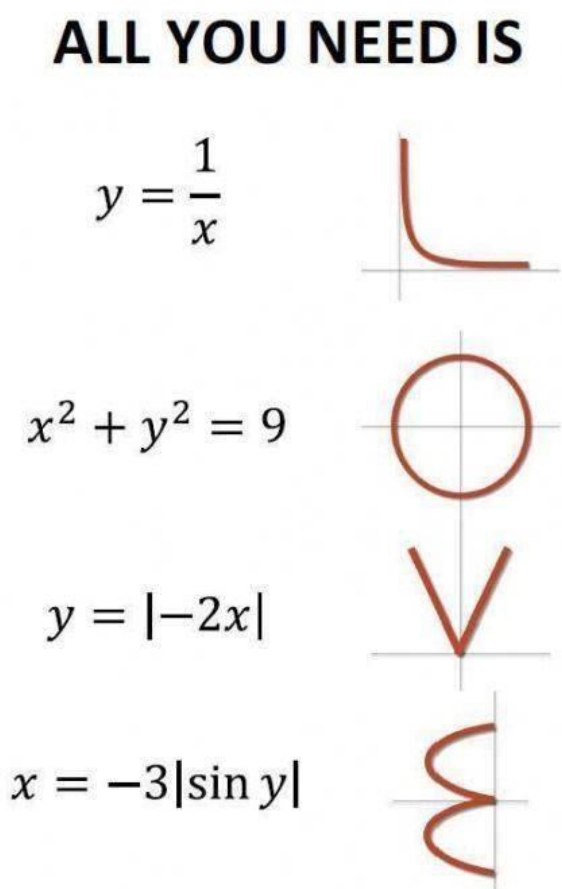
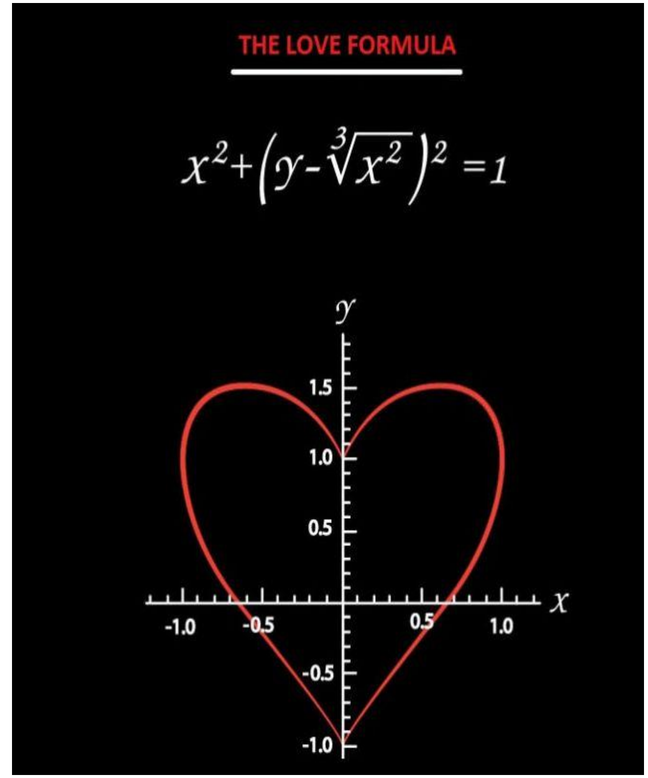
- So, our victim died around $20 : 29$.

Mystery solved!...


ALL YOU NEED IS
$y = \frac{1}{x}$
$x^2 + y^2 = 9$
$y = |-2x|$
$x = -3|\sin y|$


THE LOVE FORMULA
$x^2+\left(y-\sqrt[3]{x^2}\right)^2=1$
y
1.5
1.0
0.5
-0.5
-1.0
x
-1.0
-0.5
0.5
1.0





"Be driven by your passion! Do it for love, not for money!"

(Freeman Thomas)

Love dynamics

To arouse the interest of students for linear systems of ODEs,
Steven Strogatz (1988) discussed a simple model of the *love affairs*.

....

*"Romeo is in love with Juliet, but in our version of this story,
Juliet is a fickle lover.*

The more Romeu loves her, the more Juliet wants to run away and hide.

*But when Romeo gets discouraged and backs off, Juliet begins
to find him strangely attractive.*

*Romeo, on the other hand, tends to echo her: he warms up
when she loves him, and grows cold when she hates him."*

....

$R(t)$ = Love of Romeo (positive values of $R(t)$) /
hate for Juliet (negative values of $R(t)$) at instant t

$J(t)$ = Love of Juliet (positive values of $J(t)$) /
hate for Romeo (negative values of $J(t)$) at instant t

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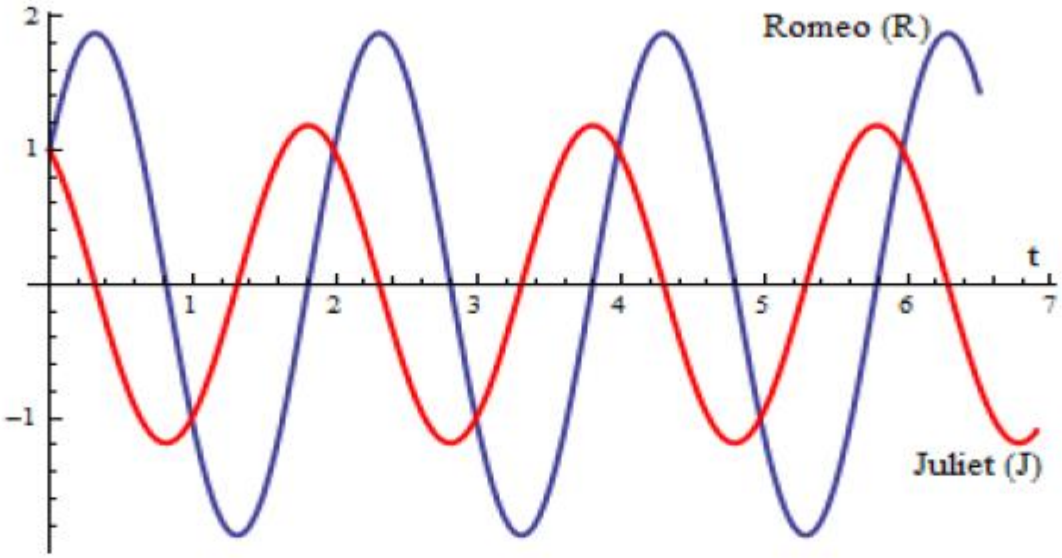
The model for this romance is

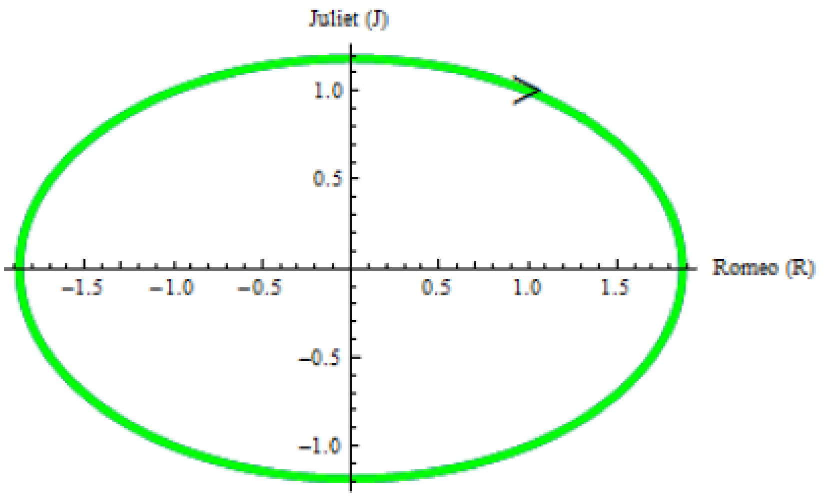
$$\begin{cases} R' = a\,J \\ J' = -b\,R \end{cases}$$

where the parameters a and b are positive, to be consistent with the story.

The temporal series



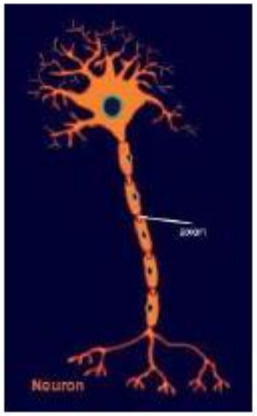
The phase space

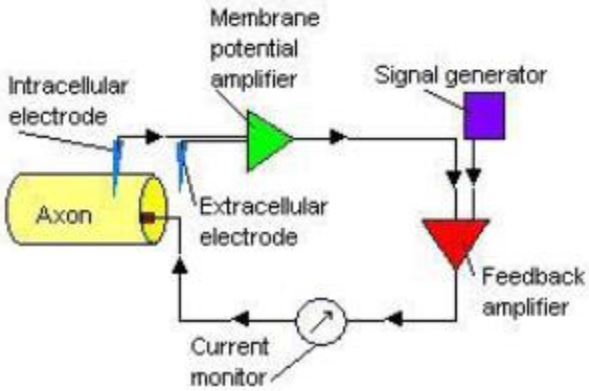


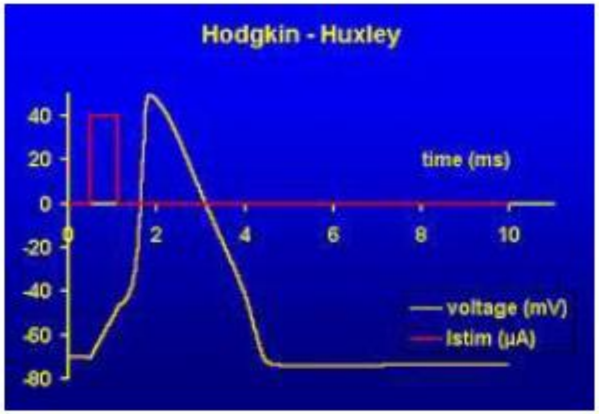
- We have a neverending cycle of love and hate.
- The governing system has a center at $(R, J) = (0, 0)$.
- At least they menage to achieve simultaneous love one-quarter of the time.

The Hodgkin-Huxley model for excitable cells

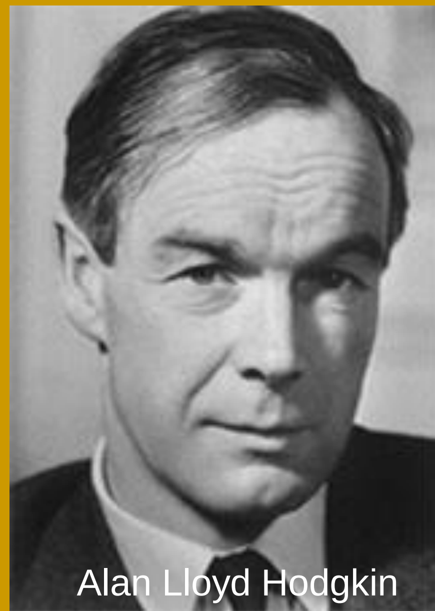
$$
\begin{cases}
C_M \frac{\partial v}{\partial t} = D_M \frac{\partial^2 v}{\partial x^2} - g_{Na} m^3 h (v - v_{Na}) - g_K n^4 (v - v_K) - g_L (v - v_L) \\
\frac{dm}{dt} = (m_\infty(v) - m)/\tau_m(v) \\
\frac{dn}{dt} = (n_\infty(v) - n)/\tau_n(v) \\
\frac{dh}{dt} = (h_\infty(v) - h)/\tau_h(v).
\end{cases}
$$

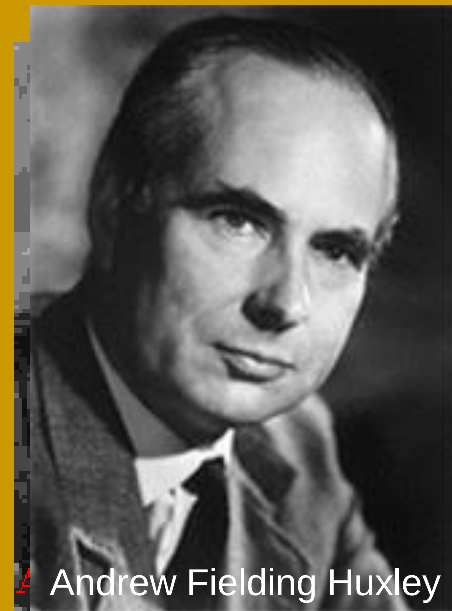






The Nobel Prize in Physiology or Medicine (1963)





"for their discoveries concerning the ionic mechanisms involved in excitation and inhibition in the peripheral and central portions of the nerve cell membrane"

Hodgkin, A.L. and A.F. Huxley, *A quantitative description of membrane current and its application to conduction and excitation in nerve,* J. Physiol. 117(1952), pp.500--544.

The FitzHugh-Nagumo model

$$\begin{cases} \dfrac{dv}{dt} = f(v) - w + I \equiv g_1(v,w) \\ \\ \dfrac{dw}{dt} = bv - \gamma w \equiv g_2(v,w) \end{cases}, \text{ with } f(v) = v(1-v)(v-a).$$

I - depolarized current

v - voltage

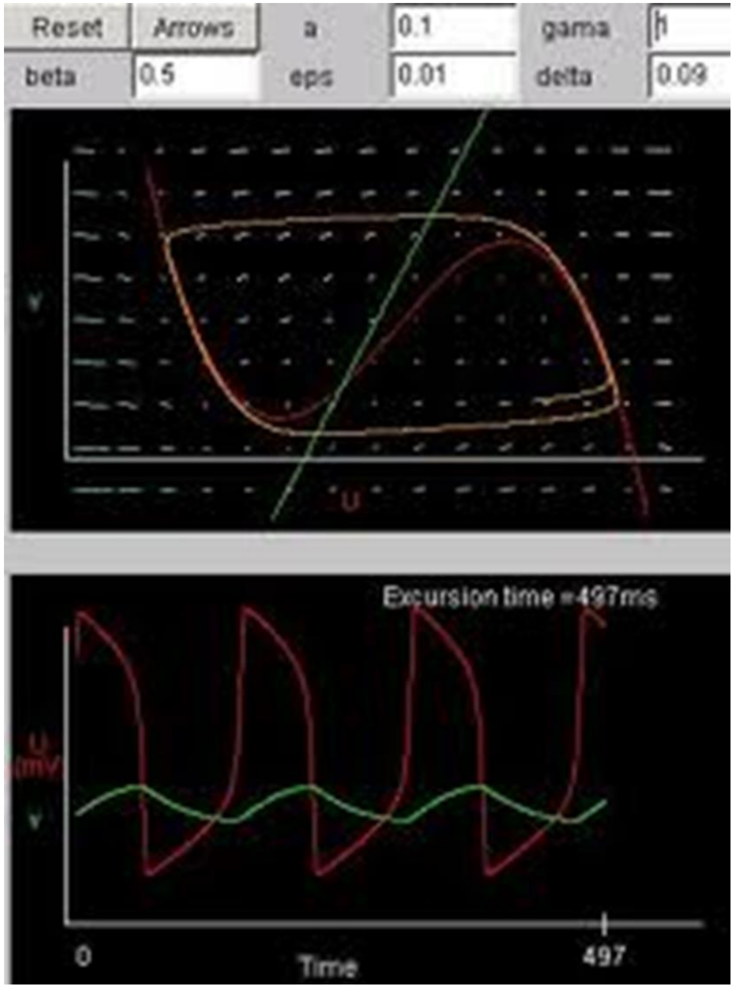
w - recovery variable

Richard FitzHugh in his Lab (National Institute of Health Maryland CA (1960)

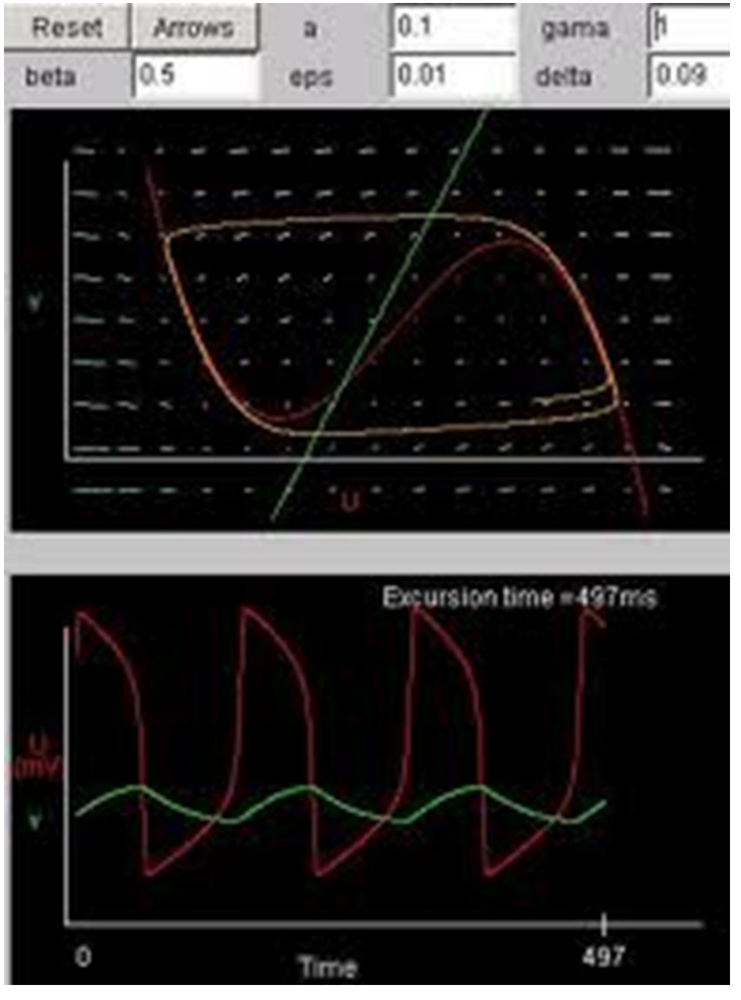
Jin-ichi Nagumo

The Nagumo's original circuit (Tokyo University)

Phase space and time series



Phase space and time series



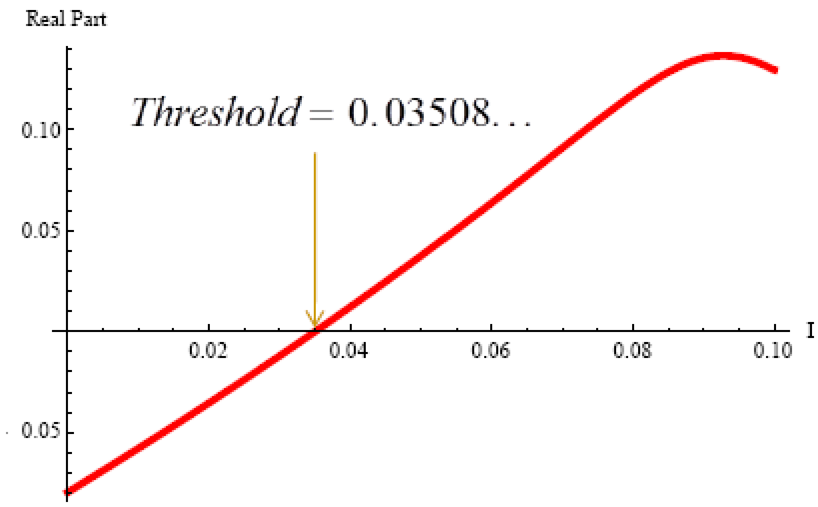
Fixed points of the system (v_e, w_e)

$$\begin{cases} \frac{dv}{dt} = 0 \\ \frac{dw}{dt} = 0 \end{cases} \Leftrightarrow \begin{cases} w = v(1-v)(v-a) \\ w = \frac{b}{\gamma} v \end{cases}$$

Linearized system

$$\begin{bmatrix} \frac{d\xi}{dt} \\ \frac{d\eta}{dt} \end{bmatrix} = \begin{bmatrix} f'(v_e) - \lambda & -1 \\ b & -\gamma - \lambda \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix}$$

What is the value of I needed to destabilize the equilibrium point of the model?



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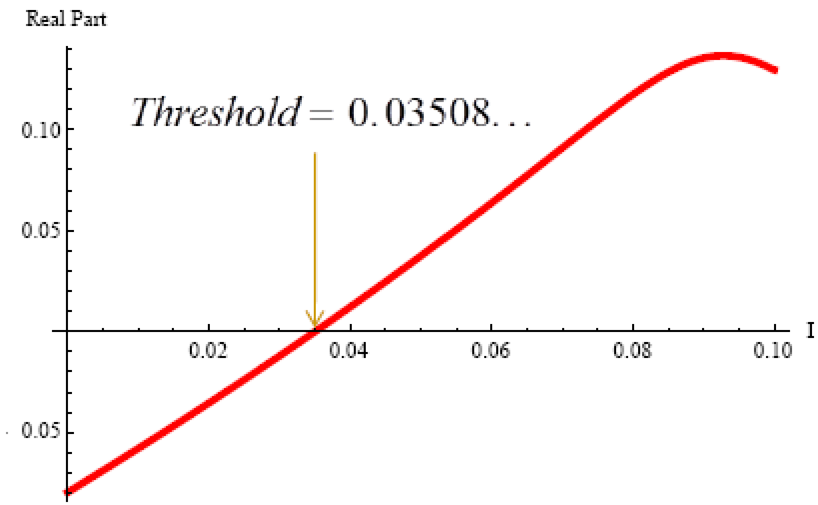
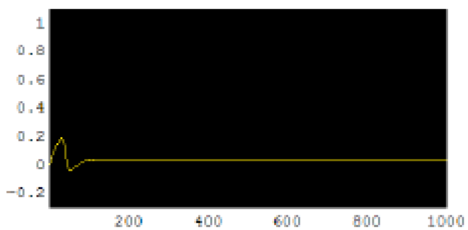


(a) $I = 0.015$



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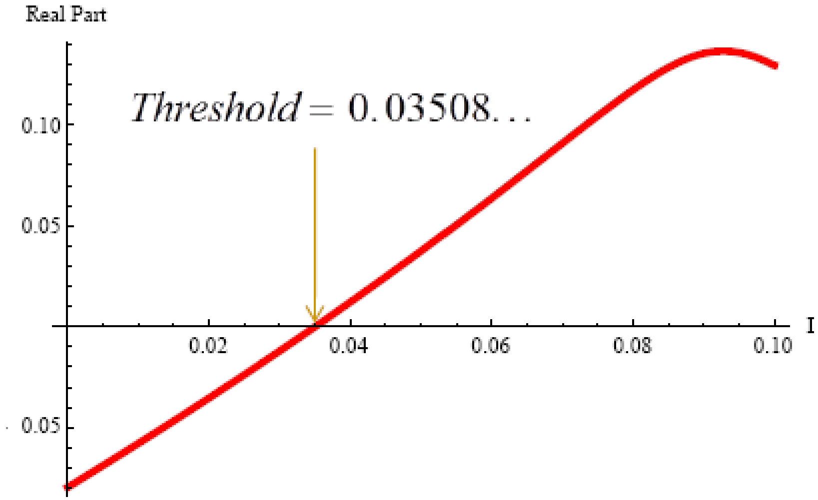

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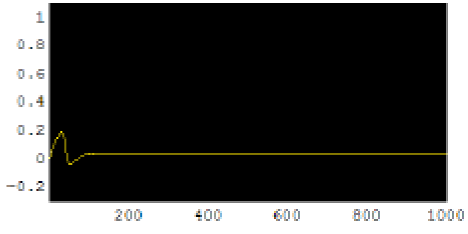
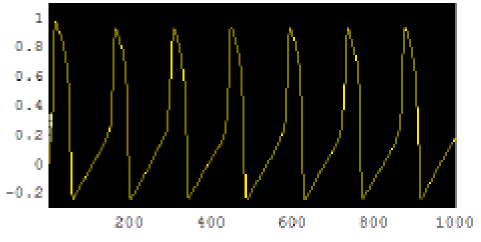


(b) $I = 0.036$



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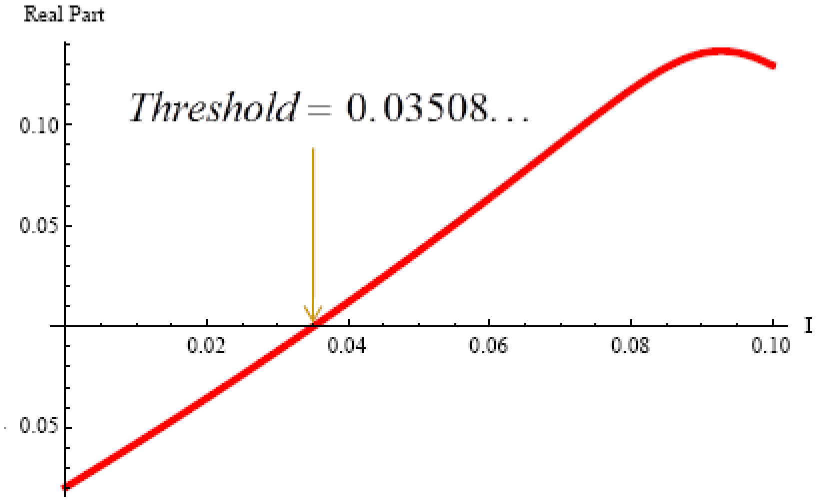
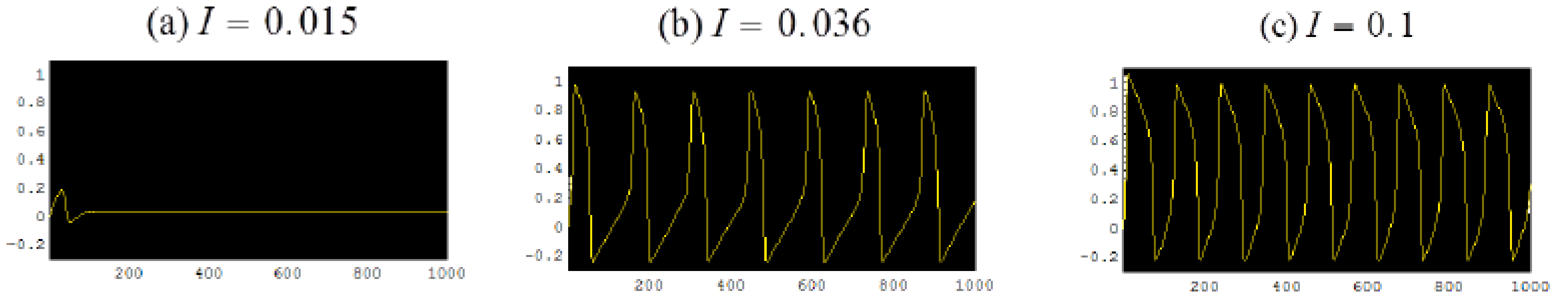
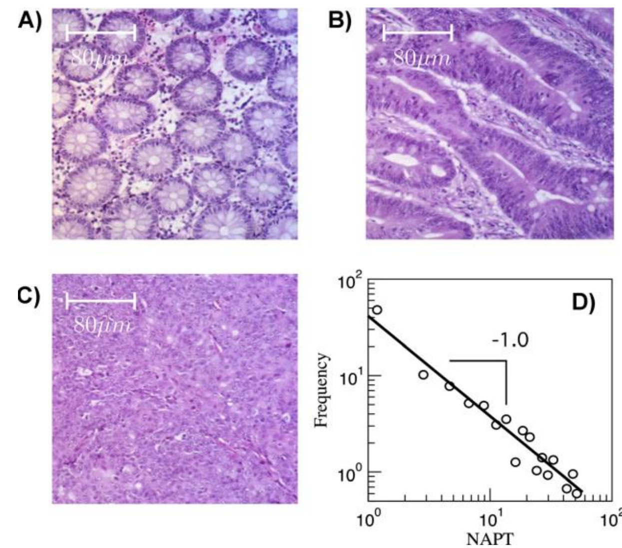




"all or nothing law"

Mathematical model of tumour growth

$$
\begin{cases}
\frac{dx_1}{dt} = x_1(1 - x_1) - a_{12}x_1x_2 - a_{13}x_1x_3, \\
\frac{dx_2}{dt} = r_2x_2(1 - x_2) - a_{21}x_1x_2, \\
\frac{dx_3}{dt} = \frac{r_3x_1x_3}{x_1 + k_3} - a_{31}x_1x_3 - d_3x_3.
\end{cases}
$$



x_1 - tumour cells

x_2 - healthy host cells

x_3 - effector immune cells

Mathematical model of tumour growth

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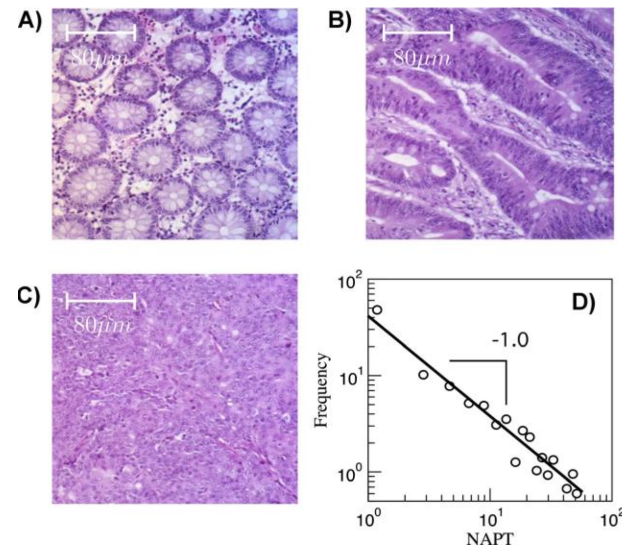
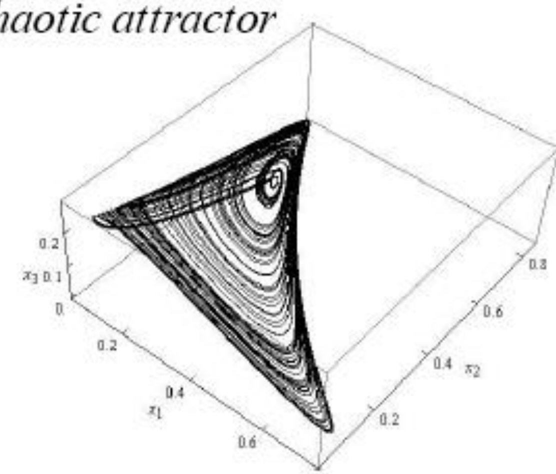


Chaotic attractor



Wonders of the nonlinear world...

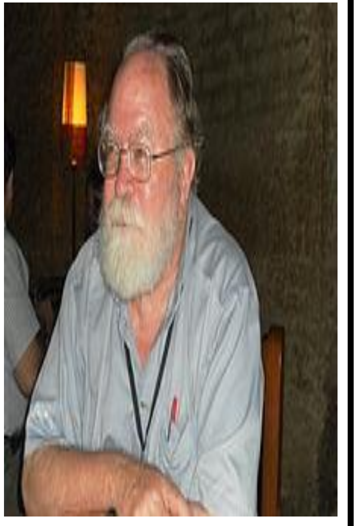
- The studied dynamical systems are sets of deterministic rules, represented by differential equations, that describe the behavior of certain magnitudes evolving in time.

- Depending on the initial conditions, and on specific choice of parameters, these dynamical variables can evolve in time towards some asymptotic behavior - *chaotic attractor*.

Dynamics - A Capsule History

1666	Newton	Invention of calculus, explanation of planetary motion
1700s		Flowering of calculus and classical mechanics
1800s		Analytical studies of planetary motion
1890s	Poincaré	Geometric approach, nightmares of chaos
1920–1950		Nonlinear oscillators in physics and engineering, invention of radio, radar, laser
1920–1960	Birkhoff Kolmogorov Arnol'd Moser	Complex behavior in Hamiltonian mechanics
1963	Lorenz	Strange attractor in simple model of convection
1970s	Ruelle &Takens	Turbulence and chaos
	May	Chaos in logistic map
	Feigenbaum	Universality and renormalization, connection between chaos and phase transitions
		Experimental studies of chaos
	Winfree	Nonlinear oscillators in biology
	Mandelbrot	Fractals
1980s		Widespread interest in chaos, fractals, oscillators, and their applications

Chaos



- The word *chaos* was coined in 1975, by Tien-Yien Li and James Yorke, to designate a long term behavior:

(i) *aperiodic*

(solutions with irregular behavior when $t \to \infty$)

(ii) in a *deterministic* system

(the irregular behavior appears from the nonlinearities of the system)

(iii) which exhibits *sensitivity to initial conditions*

(the nonlinearities amplify exponentially tiny variations of the initial conditions)

Attractor

- An **attractor** is a set to which all neighboring trajectories converge.
- A **chaotic attractor** exhibits *sensitive dependence on initial conditions*.

Attractor

- An **attractor** is a set to which all neighboring trajectories converge.
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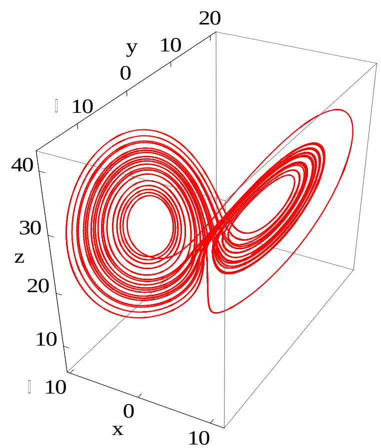
The Lorenz system

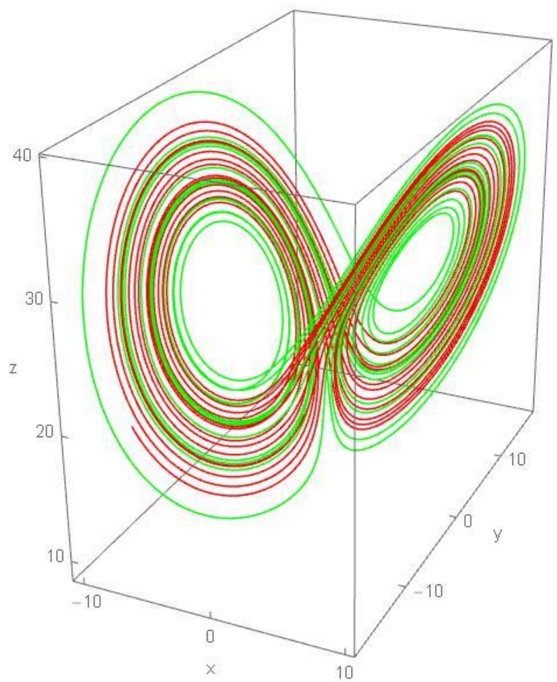
$$\begin{cases} \dfrac{dx}{dt} = \sigma(x-y) \\ \dfrac{dy}{dt} = x(\rho - z) - y \\ \dfrac{dz}{dt} = xy - \beta z \end{cases}$$

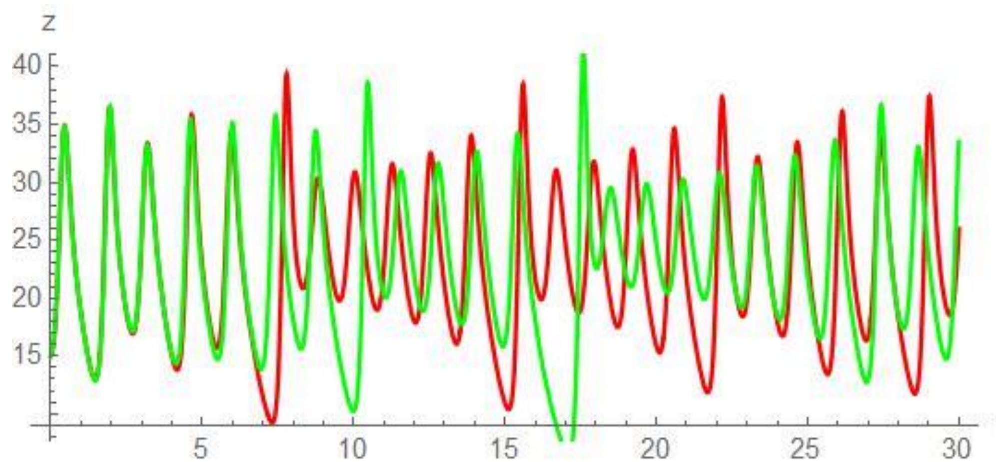


Edward N. Lorenz

(Inspired by investigations in atmospheric dynamics, 1963)

- Lorenz discovered that a wonderful structure with the dynamical property of *sensitive dependence on initial conditions* emerges, if the solution is vizualized in phase space.
- The chaotic attractor has a *butterfly pattern*.

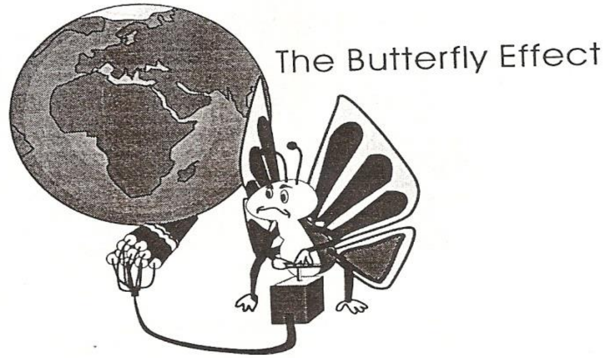


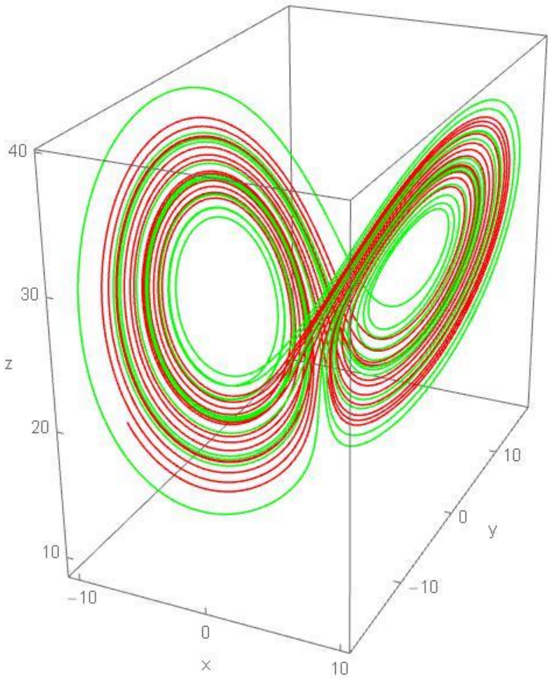


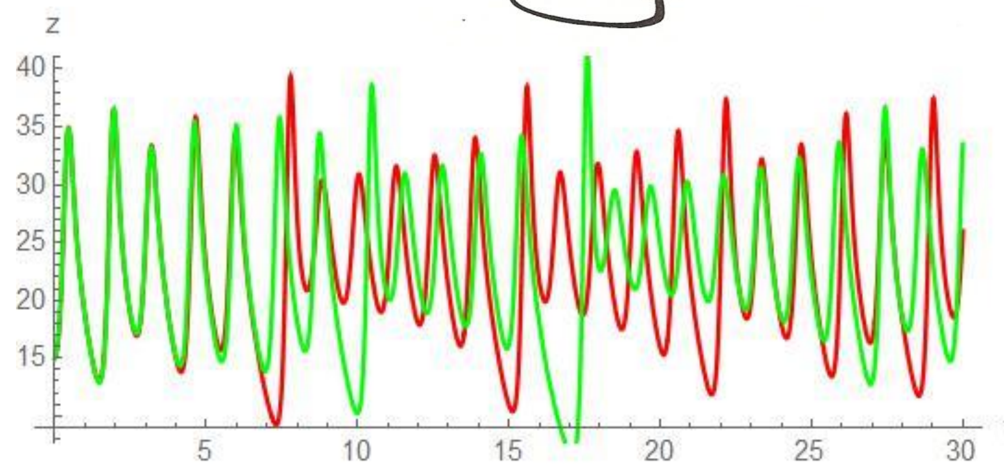
(RED) $x(0)=3.0$; $y(0)=15.0$; $z(0)=6.5$

(GREEN) $x(0)=3.05$; $y(0)=15.05$; $z(0)=6.55$

- After an initial transient, the solution settles into an *irregular* oscillation that persists as $t \to \infty$, but never repeats exactly. The motion is **aperiodic**.







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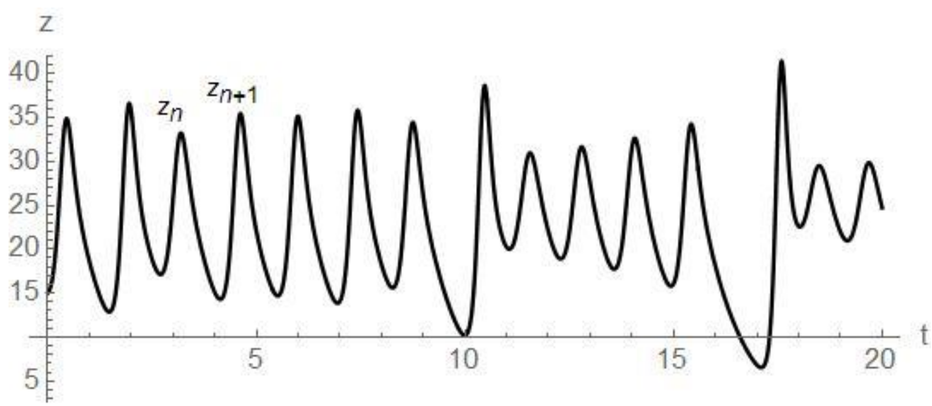
- Lorenz (1963) found a remarkable way to analyze the dynamics of a chaotic attractor. He directs our attention to a particular view of the attractor.

- Recording the successive nth local maxima of $z(t)$, in the form (z_n, z_{n+1}), we obtain



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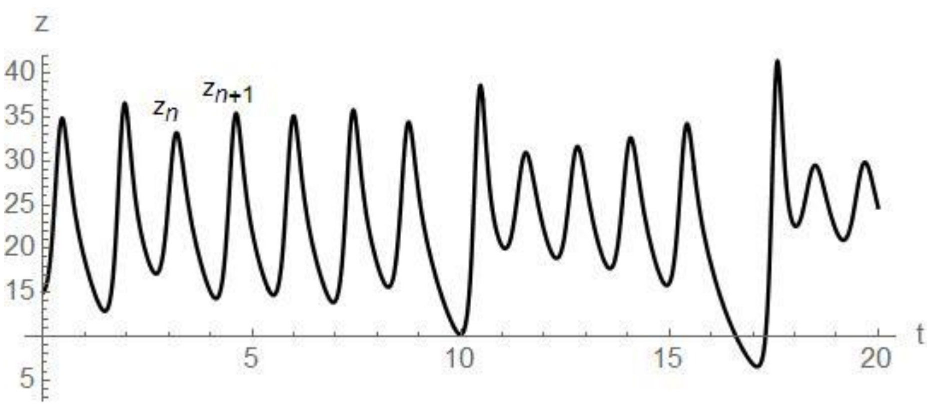
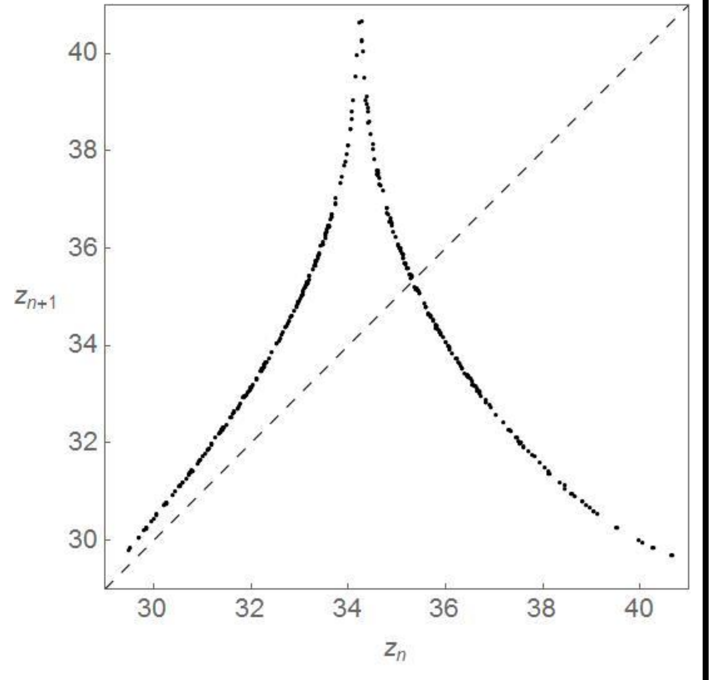
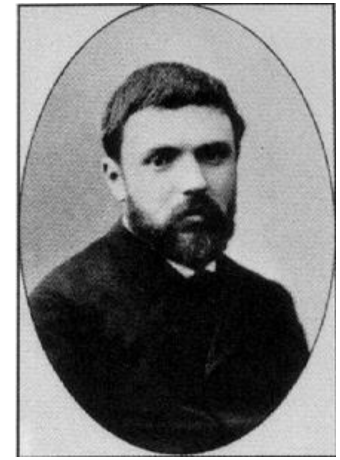
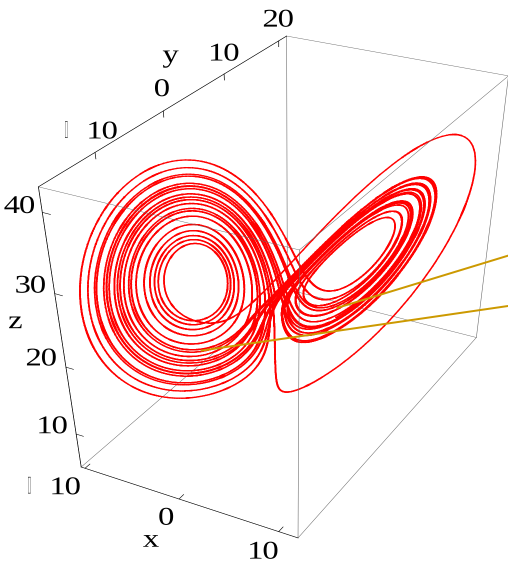


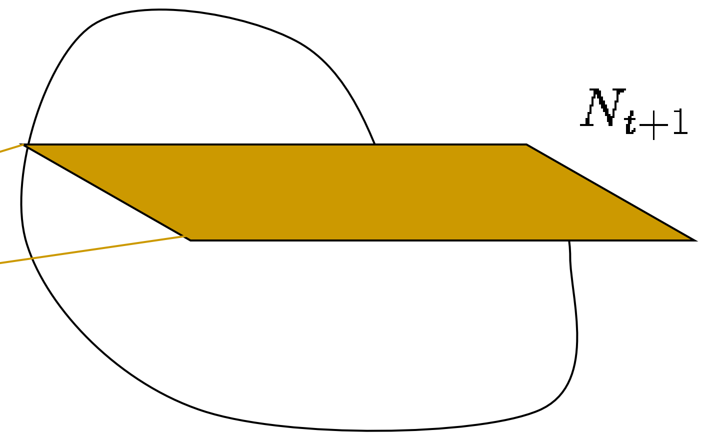


- Using a *Poincaré section* to reduce the phase space dimension, we simplify the analysis recording the visits of the 3D solution to a plane.
  Surprisingly, we obtain a map with the same features.



Poincaré, Henri (1854-1912)





$$N_{t+1} = f(N_t)$$

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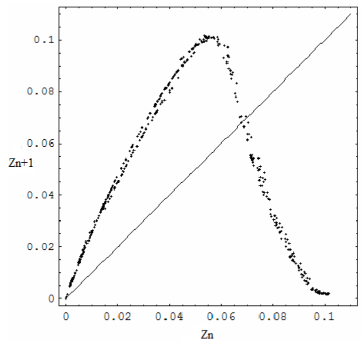

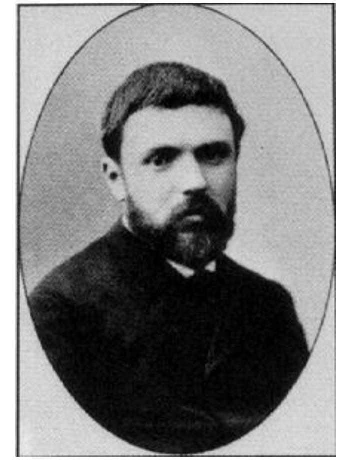

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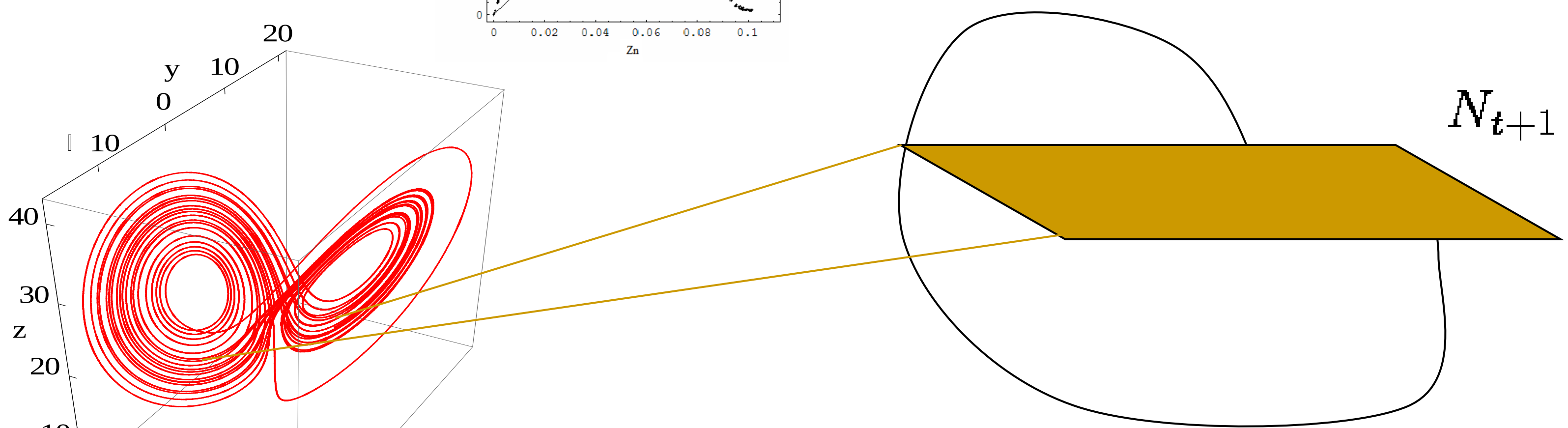


$$N_{t+1} = f(N_t)$$

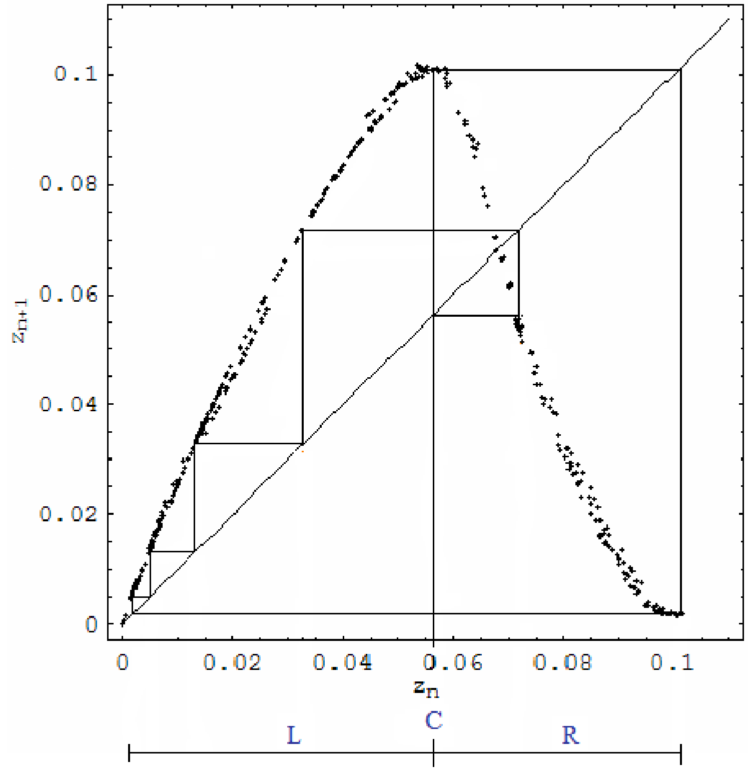
Unimodal maps

- Let us consider the map

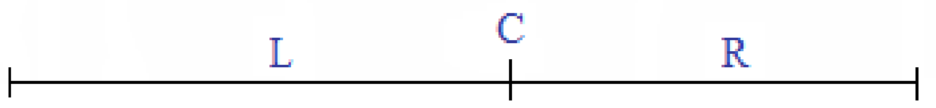
- The point *C* at which the family of maps has a maximum is called a turning point.

- The dynamics is characterized by the symbolic sequences associated to the orbit of the point *C*.

- We associate to the orbit a sequence of symbols. Each symbol corresponds to a new iteration.



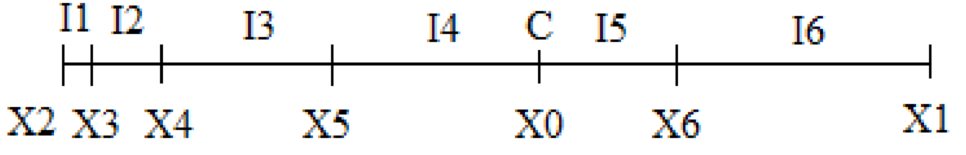
- So, we have a symbol L, C or R according to the place where the iterates of the turning point fall.



- That is, we consider a correspondence between points of the interval and symbols of an alphabet $\mathcal{A} = \{L, C, R\}$.

- The orbit of the turning point in our map defines the period-7 kneading sequence *RLLLLRC*.

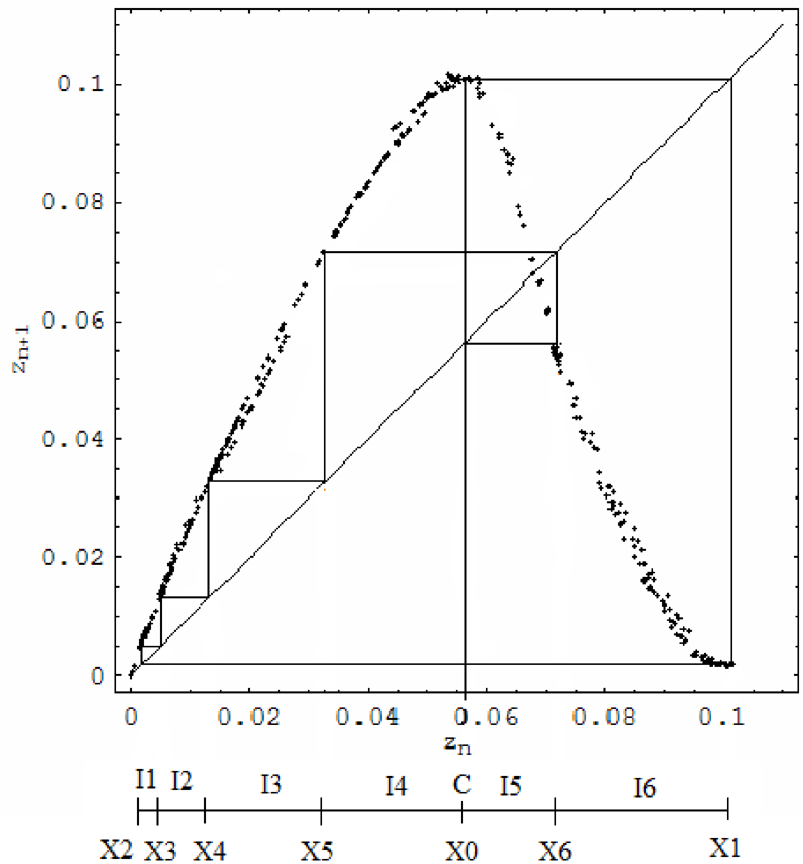
- This sequence defines the partition



$$x_2 < x_3 < x_4 < x_5 < x_0 < x_6 < x_1$$

- We can now study the transitions of the subintervals I_k

	I_1	I_2	I_3	I_4	I_5	I_6
I_1	0	1	0	0	0	0
I_2	0	0	1	0	0	0
I_3	0	0	0	1	1	0
I_4	0	0	0	0	0	1
I_5	0	0	0	0	1	1
I_6	1	1	1	1	0	0



RLLLLRC

- Therefore, the corresponding transition matrix is

$$M(f) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

wich has the characteristic polynomial

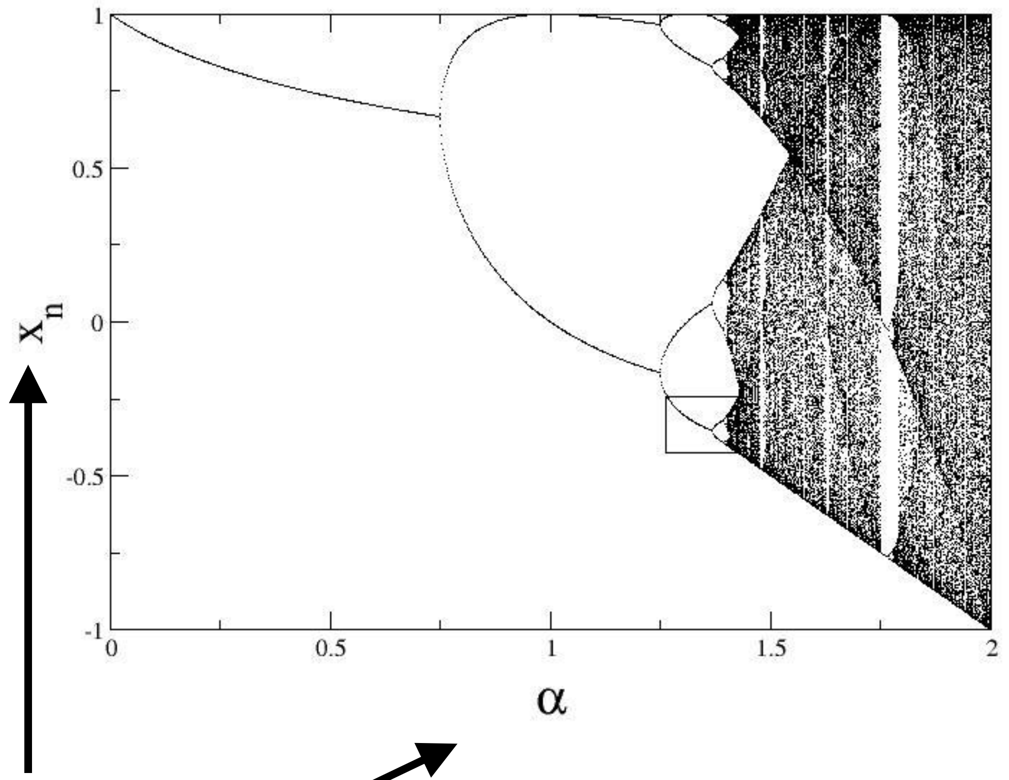
$$p(\lambda) = \det[M(f) - \lambda I] = 1 - \lambda - \lambda^2 - \lambda^3 - \lambda^4 - \lambda^5 + \lambda^6.$$

The growth number $s(f)$ (the spectral radius of matrix $M(f)$) is $1.94686\ldots$.

So, the value of the topological entropy can be given by

$$h_{top}(f) = \log s(f) = 0.675975\ldots.$$

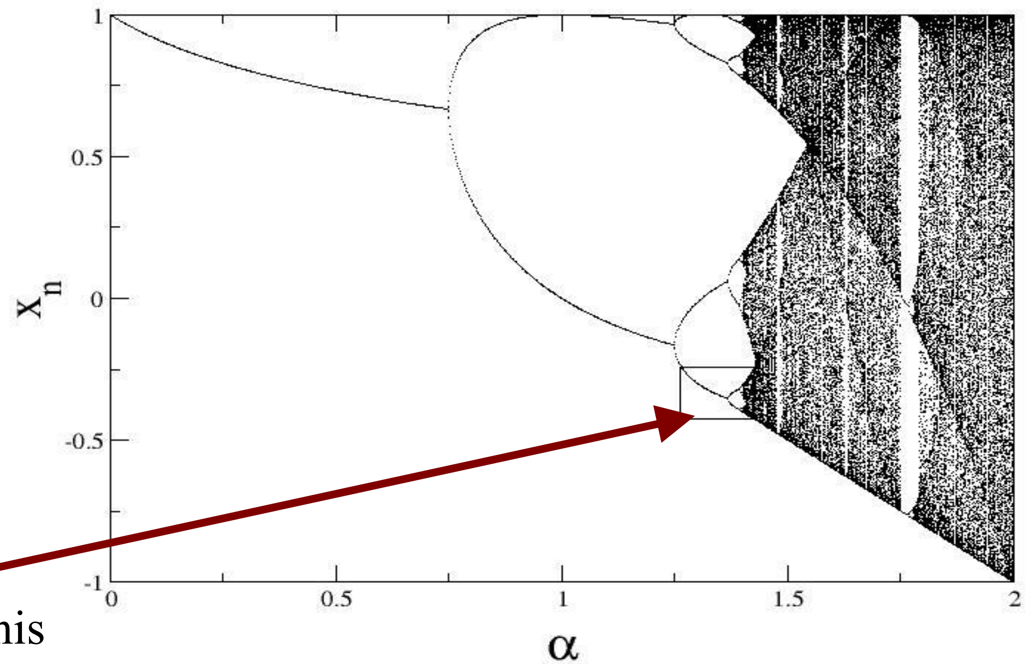
Bifurcation diagram



If we record the last 200 successive values of *x* for each value of the parameter α...

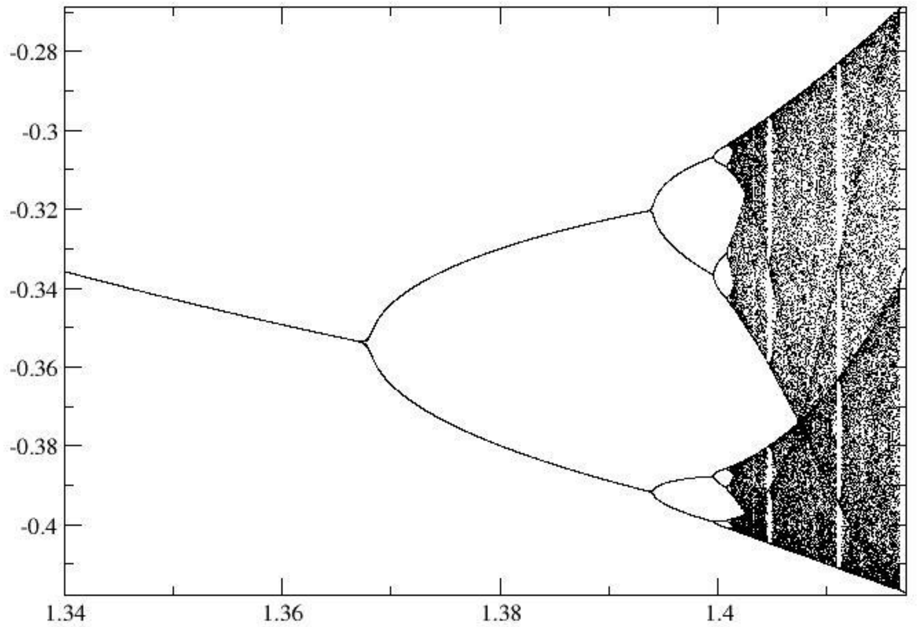
What special feature does this diagram have?

Let's take a closer look…



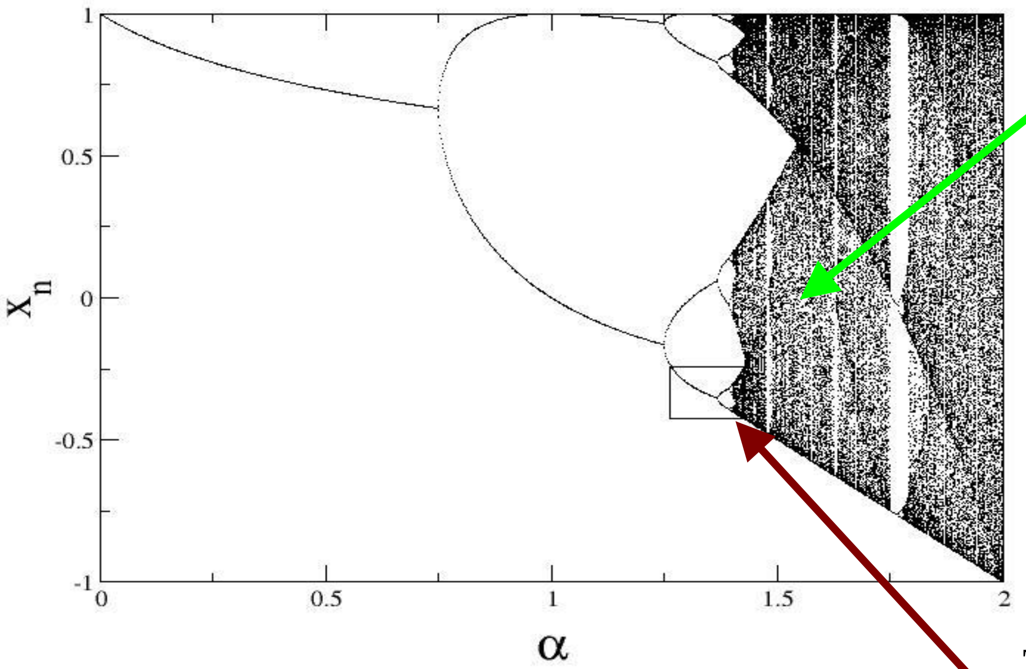
Let's amplify this region

And… We obtain something very similar!



Let's try an amplification in other region

Now, we amplify a much smaller area!

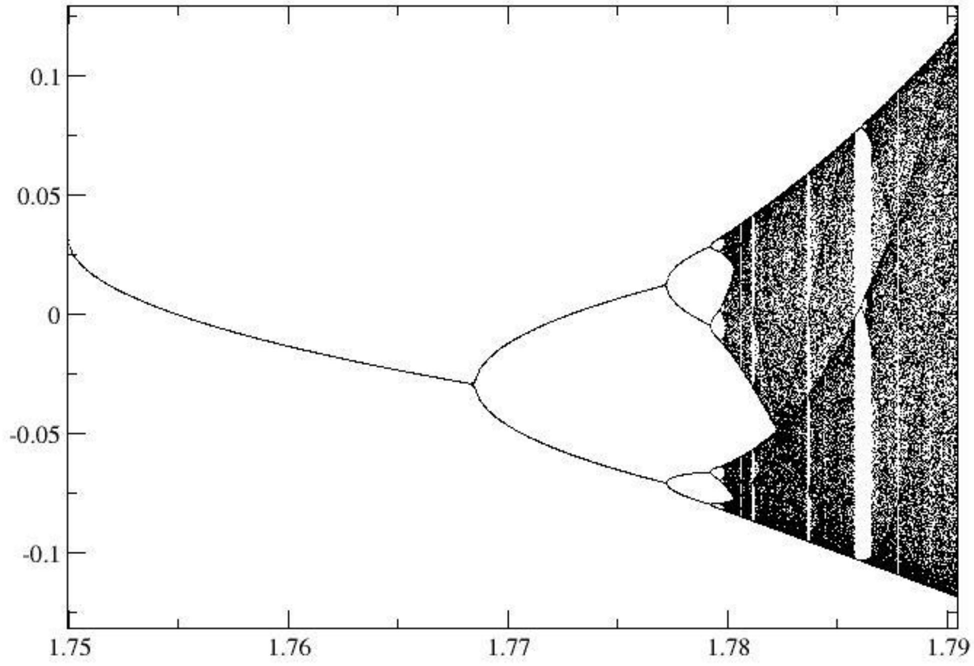


We amplify this region

Difficult to see, isn't it?

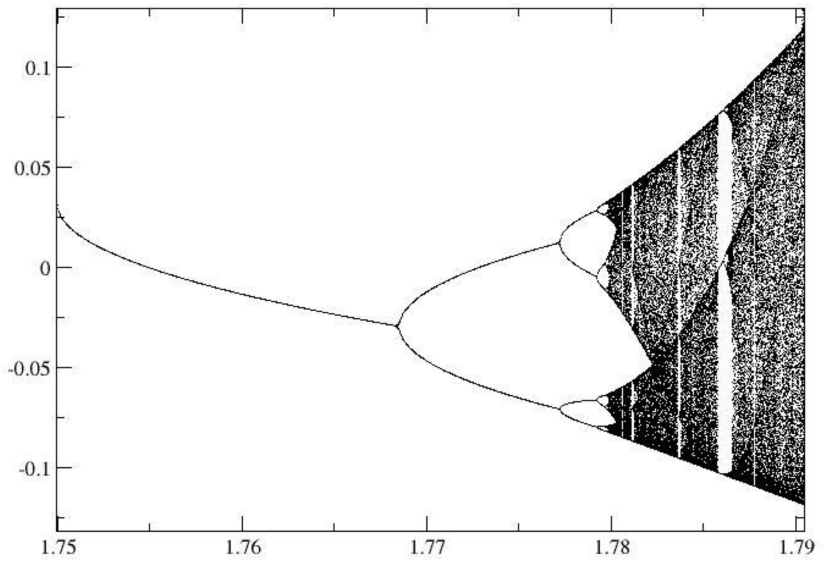
This is the region we have amplified before

The same result!



In fact, the bifurcation diagram repeats itself successively in smaller and smaller scales…

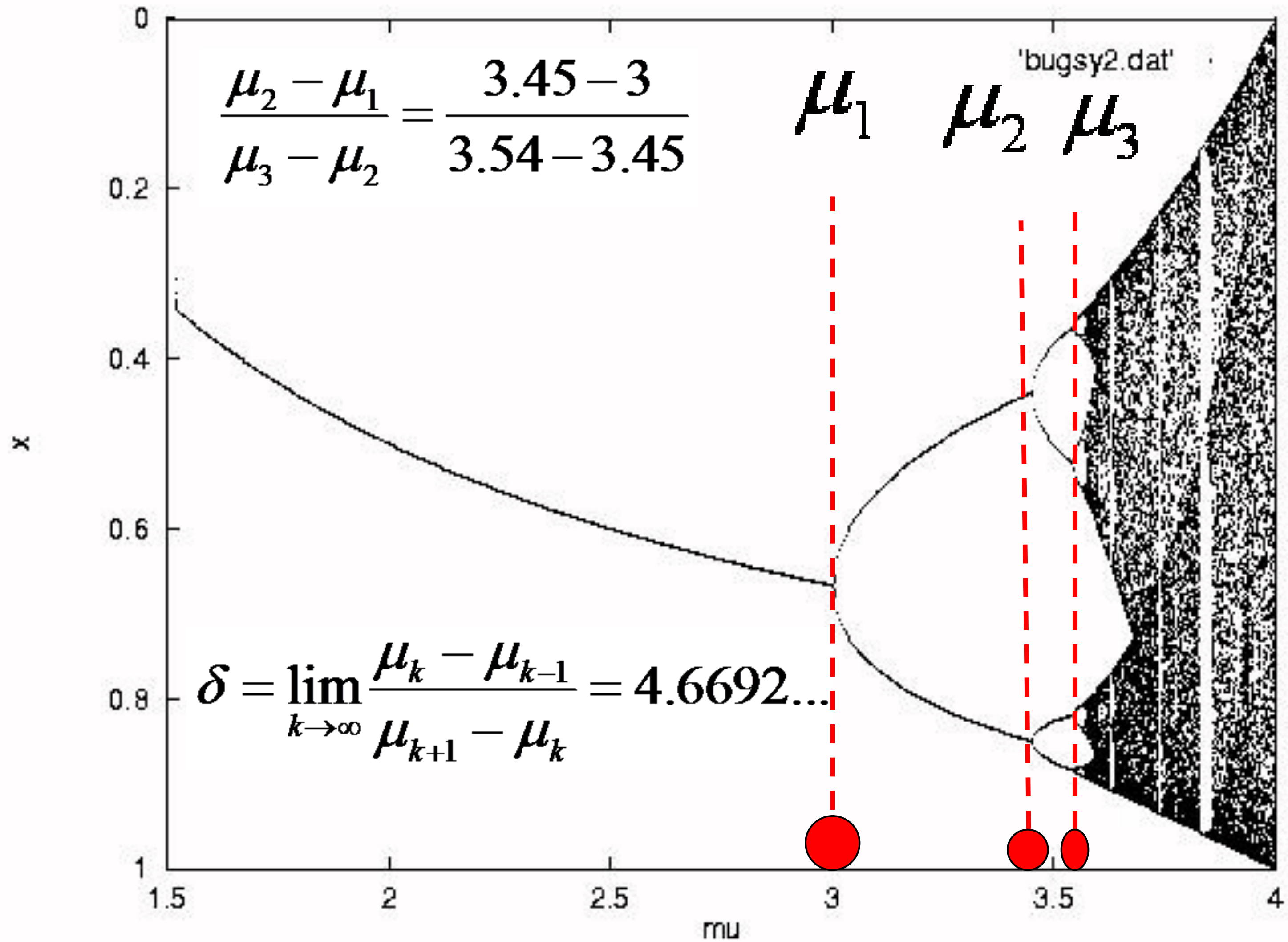
This behavior turned out to be *universal*!



There is a hidden fractal here!

Moreover, check this out!..

$$\frac{\mu_2-\mu_1}{\mu_3-\mu_2}=\frac{3.45-3}{3.54-3.45}$$



$$\delta=\lim_{k\to\infty}\frac{\mu_k-\mu_{k-1}}{\mu_{k+1}-\mu_k}=4.6692\ldots$$

- In 1975, Feigenbaum discovered this regularity, truly unexpected, in the period-doubling cascade.

$$\delta = \lim_{k\to\infty} \frac{\mu_k - \mu_{k-1}}{\mu_{k+1} - \mu_k} = 4.6692\ldots$$

- This universal value, δ, is called **Feigenbaum constant**.

Quite impressive, isn't it?

Iterated Function System (IFS) *with* a secret order…

- Let us consider a general linear vector field of the form

$$T(x,y) = \left(a\,x + b\,y + c\,,\; d\,x + e\,y + f\right) \quad \text{with} \quad a,\, b,\, c,\, d,\, e,\, f \in \mathbb{R}.$$

- We are going to consider an initial point $P_0 = (x_0, y_0)$, and a random $r \in [0,1]$ and

$$T(x,y) = \begin{cases} \left(0.05\,x, 0.2\,y\right) & \text{if } \; 0 \le r < 0.05 \\ \left(0.85\,x + 0.05\,y, -0.04\,x + 0.85\,y + 1.6\right) & \text{if } \; 0.05 \le r < 0.86 \\ \left(0.1\,x - 0.26\,y, 0.23\,x + 0.22\,y + 1.6\right) & \text{if } \; 0.86 \le r < 0.93 \\ \left(-0.15\,x + 0.28\,y, 0.226\,x + 0.24\,y + 0.44\right) & \text{if } \; 0.93 \le r \le 1 \end{cases}$$

Iterated Function System (IFS) *with* a secret order…

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Let's start with $P_0=(\overset{x}{\overset{\downarrow}{0}}.5,\overset{y}{\overset{\downarrow}{0}}.5)$

- 1^{st} iteration

For example, if $r=0.55$, we use the second branch to compute the image of the initial point $P_0=(0.5,0.5)$.

The result is $T(0.5,0.5)=(0.45,2.005)$.

Consequently, we represent in the plane the point $P_1=(0.45,2.005)$.

Iterated Function System (IFS) *with* a secret order…

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- 2^{st} iteration

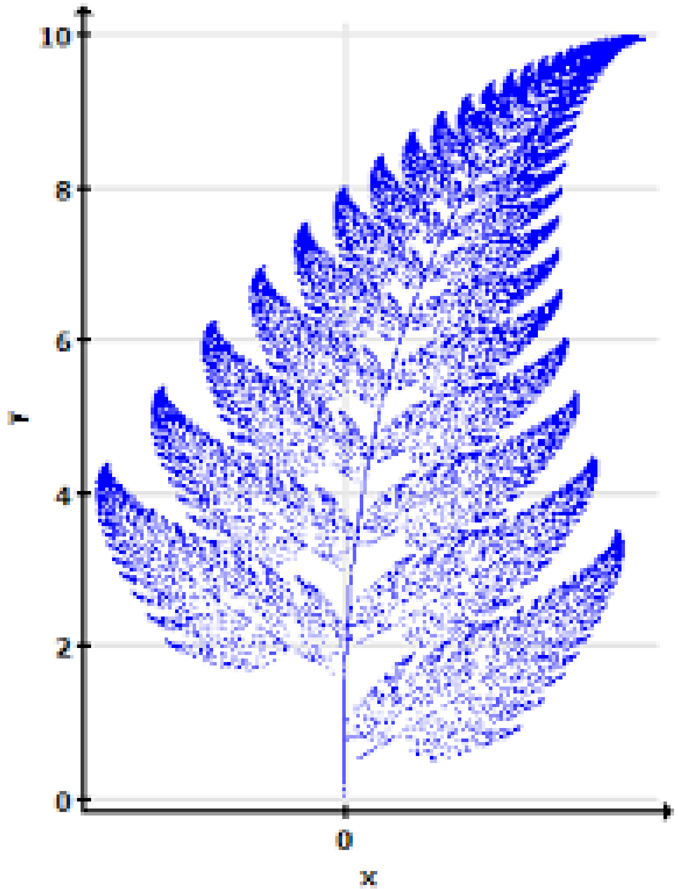
For example, if $r = 0.9$, we use the third branch to compute the image of the previous point $P_1 = (0.45, 2.005)$.

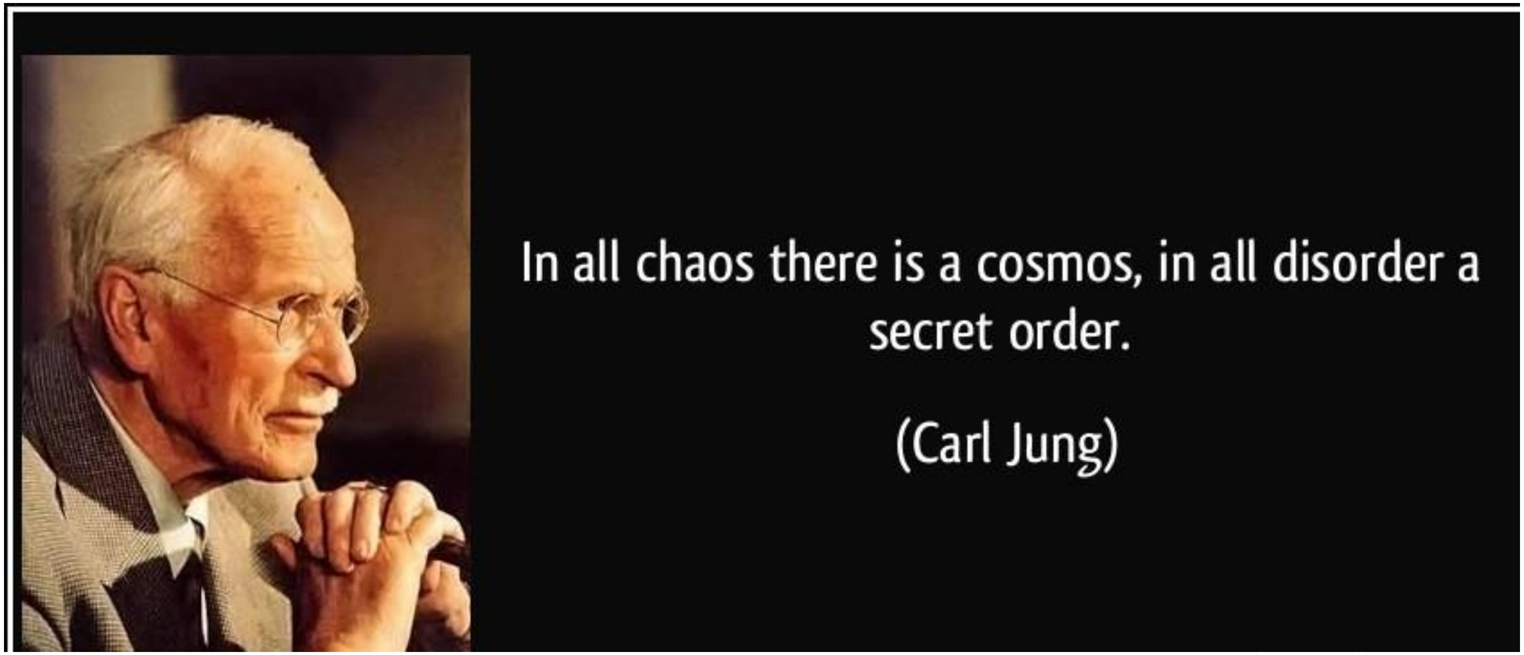
The result is $T(0.45, 2.005) = (-0.4763, 2.1446)$

Consequently, we represent in the plane the point $\boxed{P_2 = (-0.4763, 2.1446)}$.

Iterated Function System (IFS) *with* a secret order…

- Following this procedure, we obtain the so-called **Barnsley's fern**.

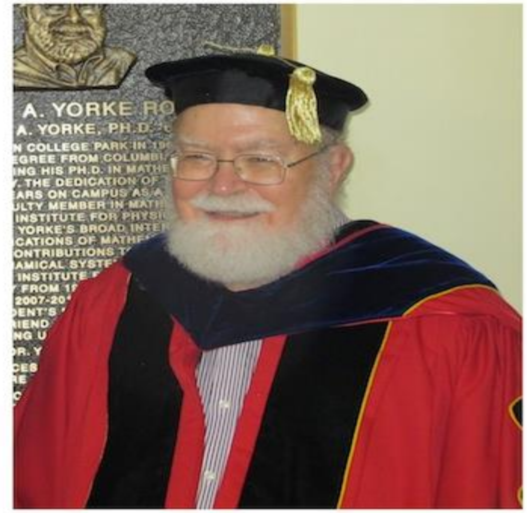



In all chaos there is a cosmos, in all disorder a secret order.
(Carl Jung)

"It is the nature of chaotic systems to surprise."

*"The **most successful people***
are those who are good at plan B."

"…this means that life can plan ahead, but
you have to be prepared to change plans .
It is a basic principle of chaos."



Prof. James A. Yorke
University of Maryland at College Park

Doctor Honoris Causa, January 28th, 2014
(Universidad Rey Juan Carlos)

And an advice offered to new generations of scientists…

"It is not the same to be a good student
and to be a good researcher,
often the two come into conflict.

*You're a **good student** if you do what you are told to,*
*while you will be a **good researcher***
if you seek to find what you do not understand."

Prof. Miguel A. Sanjuán
(Universidad Rey Juan Carlos)

Mathematics in use
(some aspects)

Analysis and application of different concepts

- **Boundedness** of the dynamical orbits

$\Psi = x_1 + x_2 + x_3$

$\frac{d\Psi}{dt} + \varepsilon\Psi \leq L.$

$\Psi(t) \leq \Psi(0)e^{-\varepsilon t} + \frac{L}{\varepsilon}(1 - e^{-\varepsilon t}) \leq \max(\frac{L}{\varepsilon}, \Psi(0)).$

- **Observability**

The *observability indices* estimate the coupling complexity

$$\delta_s = \frac{1}{T}\sum_{t=0}^{T} \frac{|\lambda_{\min}[O_s^T O_s, \mathbf{x}(t)]|}{|\lambda_{\max}[O_s^T O_s, \mathbf{x}(t)]|},$$

$\delta_{x_1} = 0.015\ldots$, $\delta_{x_2} = 0.021\ldots$ and $\delta_{x_3} = 0.0001\ldots$

The variables can be ranked in descending degree of observability

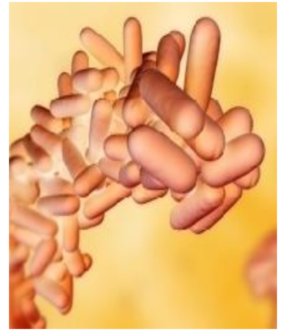
$x_2 \rhd x_1 \rhd x_3$

- **Topological entropy**
- **Lyapunov exponents**
- **Predictability**

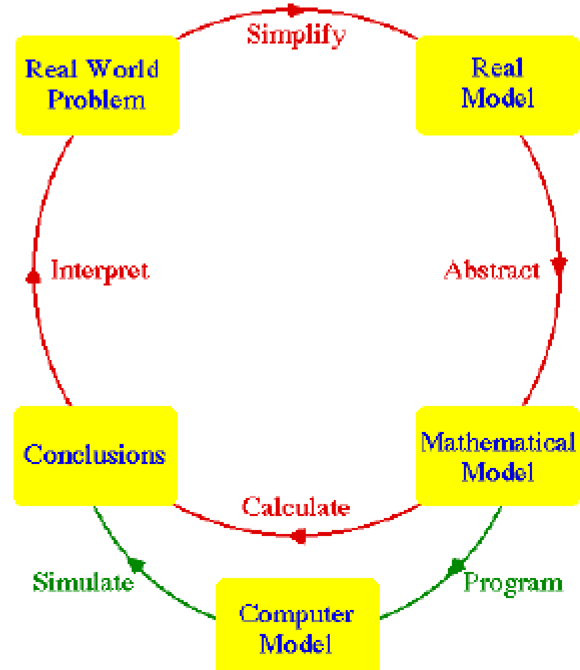
Analytical methods for highly nonlinear problems of ODEs and PDEs

- **Homotopy analysis method**
- $\left(\frac{G'}{G}\right)$ **– Expansion method**

⋮



Mathematical modelling cycle ↗



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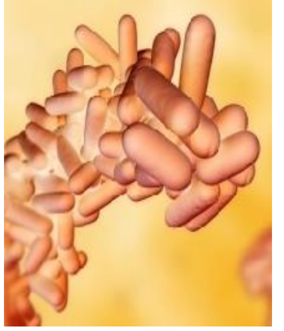
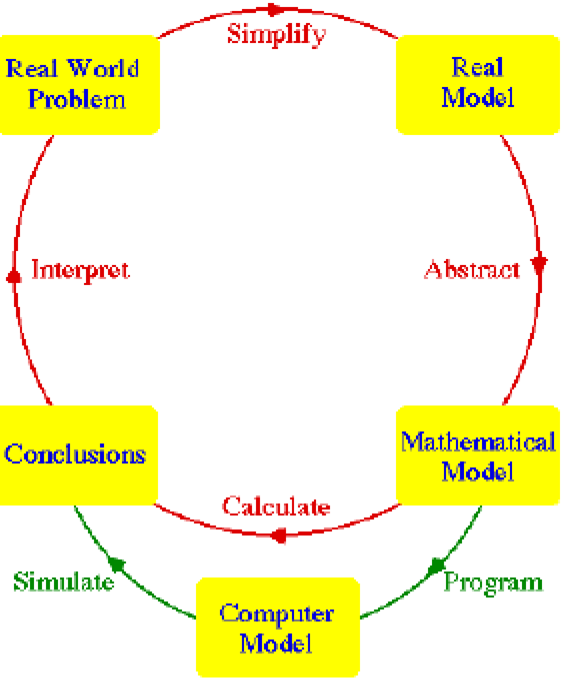




Mathematical modelling cycle ↗

The use of maths helps us…

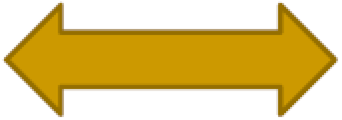
▹ to **predict** the dynamical behaviour of biological systems

▹ to **improve/create** realistic models.

▹ to **enhance our understanding** of infections and treatment processes allowing computer simulation of mechanisms which are difficult to monitor in vivo.

Research work…

…**collaborative** and **interdisciplinary**



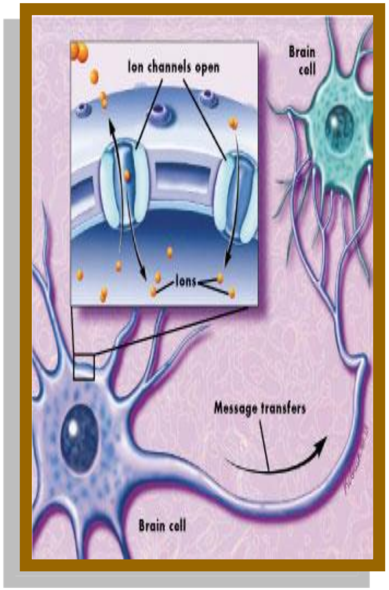
Interaction with biology

↓

realism

↓

significance of the theoretical results



Network of collaborators

Biomedical Research Park - Barcelona

Evolutionary Systems Virology Group - Valencia

Josep Sardanyés

Santiago Elena

Collaboration is key!...

N. Martins

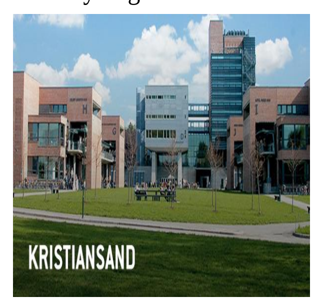
C. Januário

J. Duarte

Svitlana and Yuriy Rogovchenko

IST - Lisbon

ISEL - Lisbon



Univ. of Agder - Kristiansand

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- Along the history of science, nature has been splitted into many parts for better understanding.

- The keyword in the science of the early twenty-first century is **multidisciplinary**.

- Along the history of science, nature has been splitted into many parts for better understanding.

- **Now**, the **different areas** *must* **interact to** complete their descriptions and better **comprehend the surrounding reality**.

- **Every time that science sheds light into nature**, **revealing its hidden shapes**, **the shadow of our ignorance gets longer**.

“Nature imagination far surpasses our own.”

(Richard P. Feynman, *The character of physical law*)

Teaching / research – a multidisciplinary approach

▹ **Teaching** and working **interaction with students** are definitely major and compelling **reasons for** pursuing **an academic career**.

▹ My academic qualifications and some years of experience make me agree with the famous quote of E. T. Bell

- '**Obvious**' **is the most dangerous word in Mathematics**.

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  - ‘**Obvious**’ **is the most dangerous word in Mathematics**.

- ▹ We must be always very **careful with the level of motivation and curiosity of our students**, especially at the undergraduate level.

- ▹ When **designing a mathematical course** in Math Bio Education, do it **FOR** and **WITH** biologists.

Teaching / research – a multidisciplinary approach

▹ **Mathematics**, in particular **the study of differential equations**, **gives students an holistic perspective** which integrates in a special way key areas such as: *calculus* and *numerical methods*, *algebra*, *geometry*.

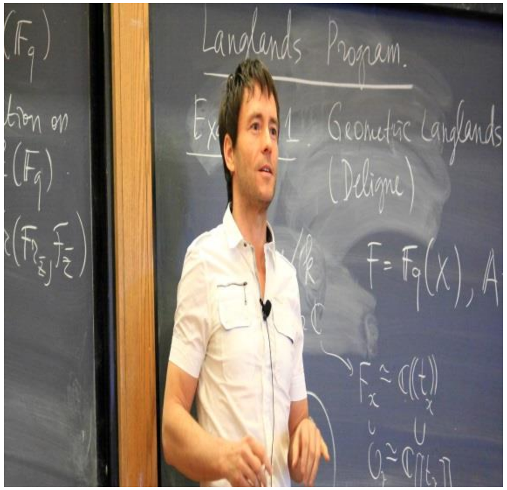
▹ One of the major benefits of **nonlinear dynamics** is that it can be used **in a wide variety of scientifically relevant situations**.

▹ It's truly important for a **professor** to have **the willingness to learn**, **pay attentio**n, **and change**.

Unique and distinctive features of Maths…

▹ **One thing should be clear**:
While our perception of the physical world can always be distorted, our perception of the **mathematical truths** can't be.
They **are objective**, **persistent**, **necessary truths**.

▹ A **mathematical formula means the same thing to anyone anywhere** - no matter what gender, ethnicity, religion; it will mean the same thing to anyone a thousand year from now. And that's why **mathematics is going to play an increasingly important role** in science and technology.



(Edward Frenkel, author of *Love & Mathematics*)

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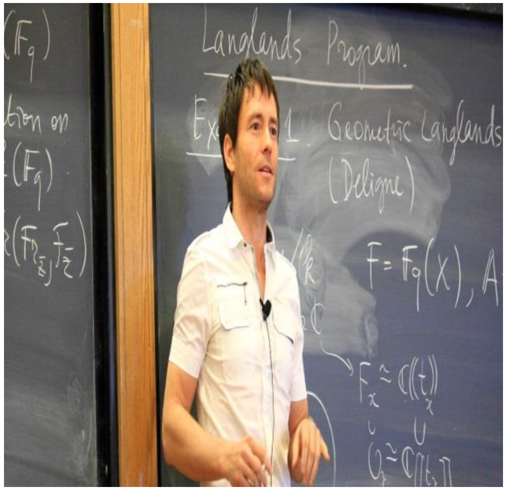
▹ It might still be possible **to be** "**bad in math**" and be a good scientist - in some areas, and probably not for too long.
But this **is a handicap and nothing to be proud of**.

▹ Granted, some areas of science currently use less math than others. But then professionals in those fields stand to benefit even more from learning mathematics.



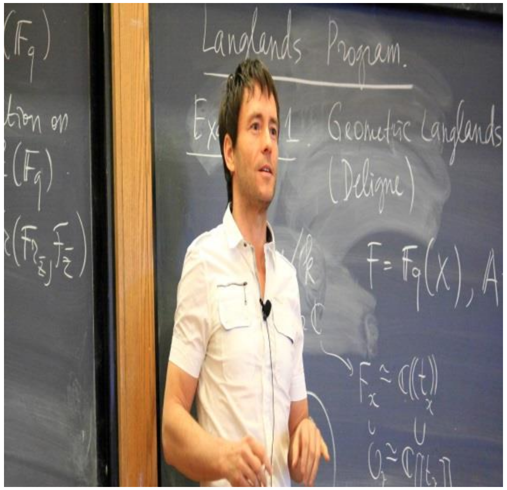
(Edward Frenkel, author of *Love & Mathematics*)

Unique and distinctive features of Maths…

▹ We should discuss the real question, which is **how to improve our math education** and to eradicate *the fear of mathematics*.

▹ *The fear of Mathematics*, among the next generation and future scientists, it's not just counterproductive;

it is a *DISGRACE, SELF-EXTINGUISHING strategy!*



(Edward Frenkel, author of *Love & Mathematics*)

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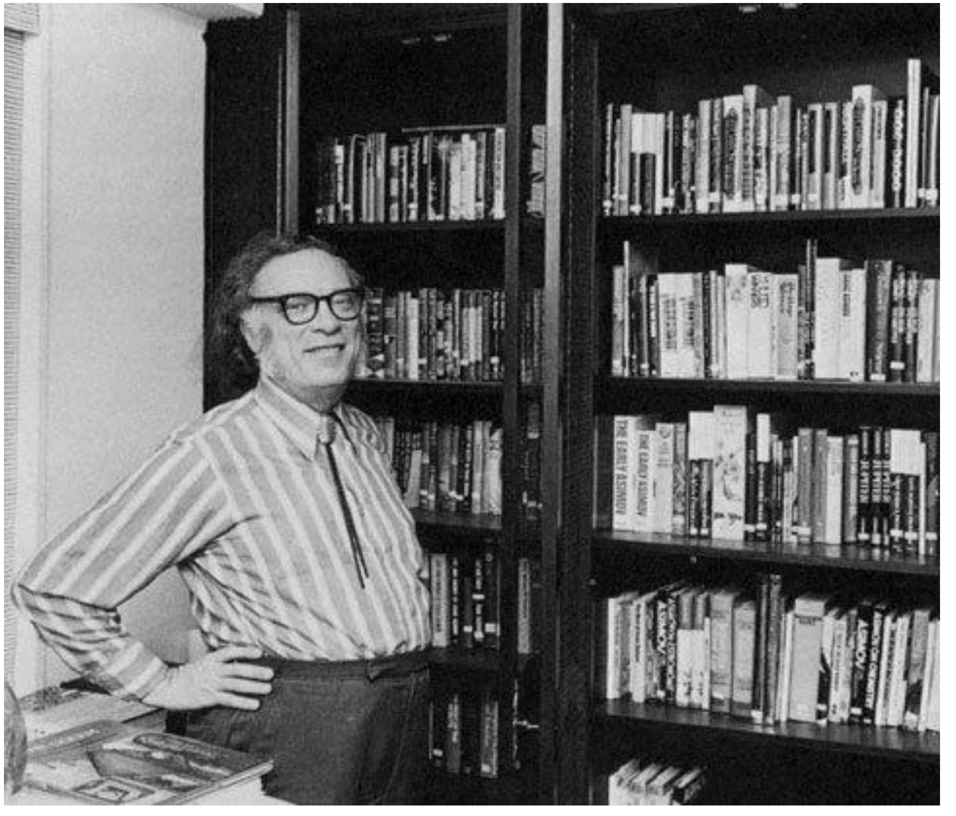

"Call it the satisfaction of curiosity. I understand a little of it today, perhaps a little more tomorrow. That's a victory in a way."

(Isaac Asimov, Profession)

Tusen takk!

Jorge Duarte (jduarte@adm.isel.pt)

Questions?

Comments?